Sub-Interval Analysis.

Incomplete Information

Alexander Harin
aaharin@yandex.ru

As the main source, see the E-Book "Introduction to Sub-Interval Analysis and its Applications" in Open Access at

http://ideas.repec.org/b/zbw/esmono/62286.html

27 September, 19.30.

Auditorium 515.

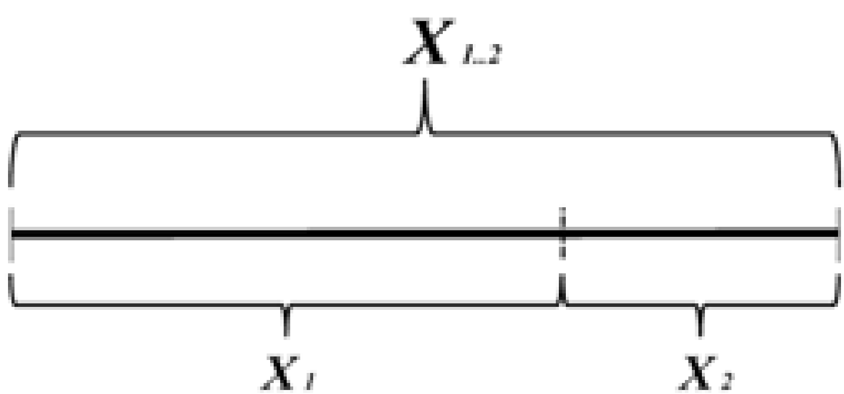
Seminar

Sub-Interval Analysis. Detailed introduction

What is It?

What is a Sub-Interval?

A sub-interval is simply
a part of an interval.



You may see an example
of an interval $X_{1..2}$ and its sub-intervals X_1 and X_2
in the above Figure.

What is It?

What is a Sub-Interval Analysis?

The sub-interval analysis is based on a very simple and clear idea:

The sub-interval analysis

is a tool to calculate

the characteristics of **a whole interval**

by means of characteristics **of its sub-intervals**.

What for is It?

What for is
the Sub-Interval Analysis?

1) To treat Inexact Data

2) To treat Incomplete Data

3) To treat Numerous Data

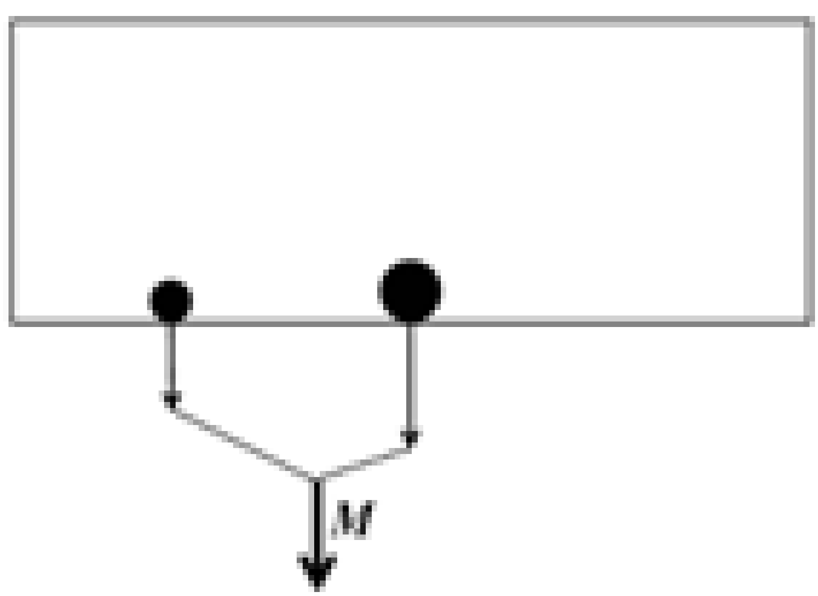
What for is It?

Where is the weight?

Sub-Boxes treat Inexact Data

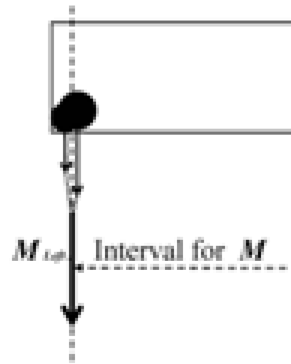
An example of balls in a box.

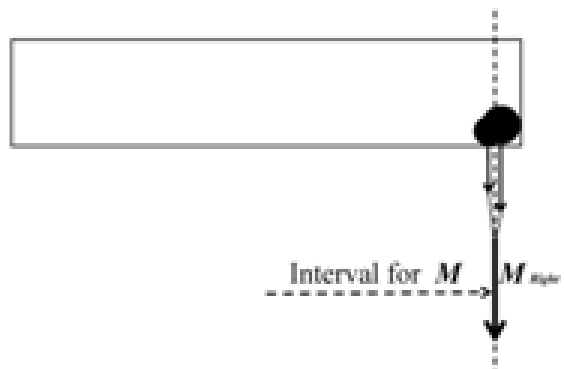
Box without Sub-Boxes 1.

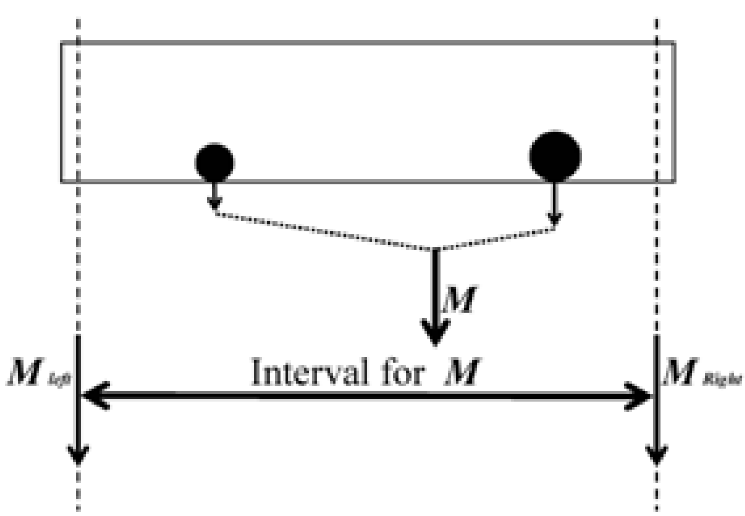


Where is the weight?

Box without Sub-Boxes 2.

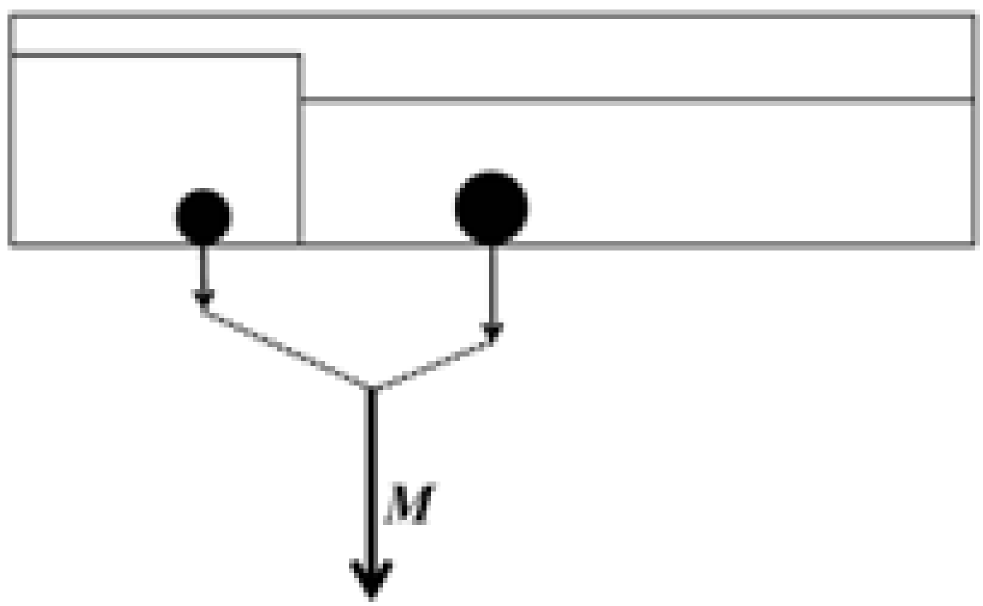






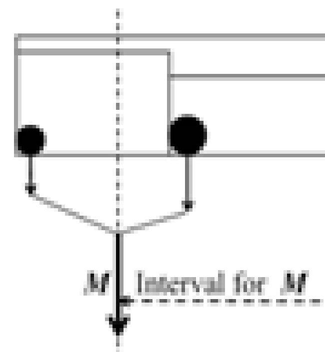
Where is the weight?

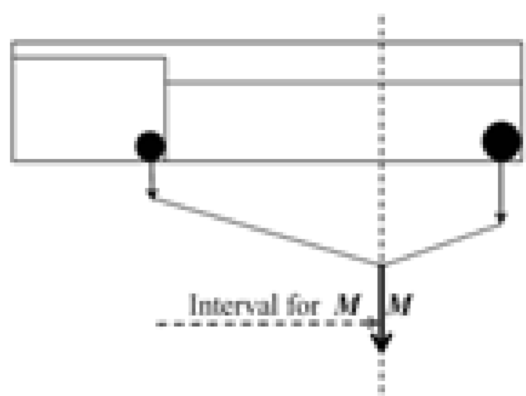
Box with two Sub-Boxes 1.

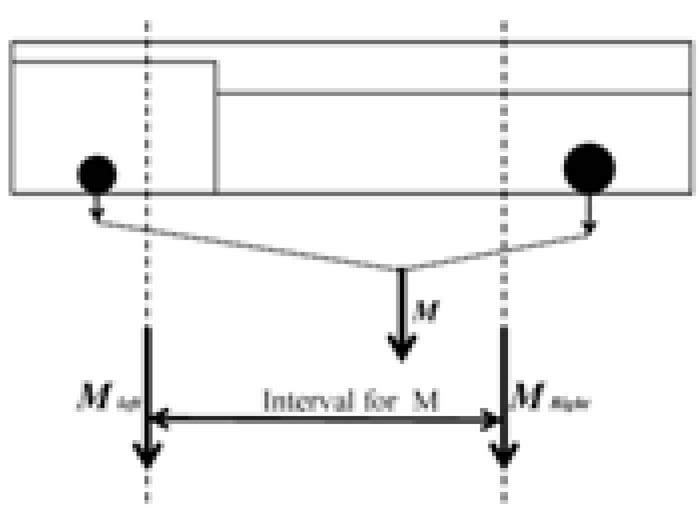


Where is the weight?

Box with two Sub-Boxes 2.

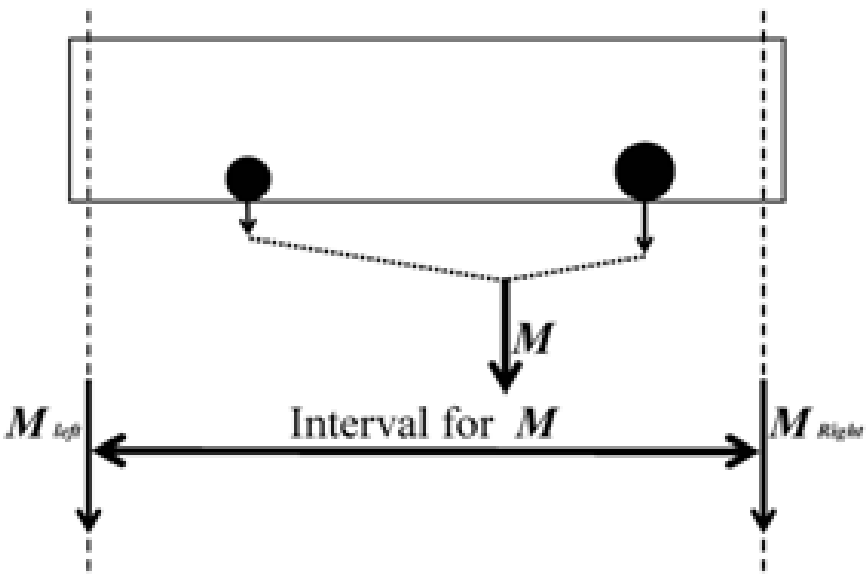


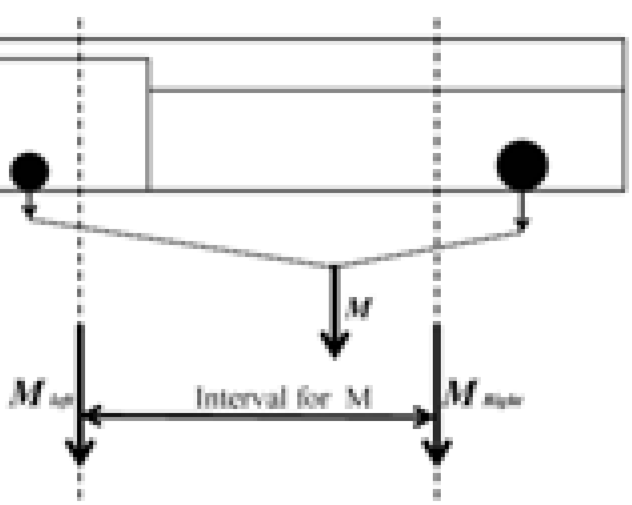




Where is the weight?

Sub-Boxes treat Inexact Data
Sub-Boxes improve the Precision





First General Results
of Sub-Interval Analysis
as a New Branch of Interval Analysis

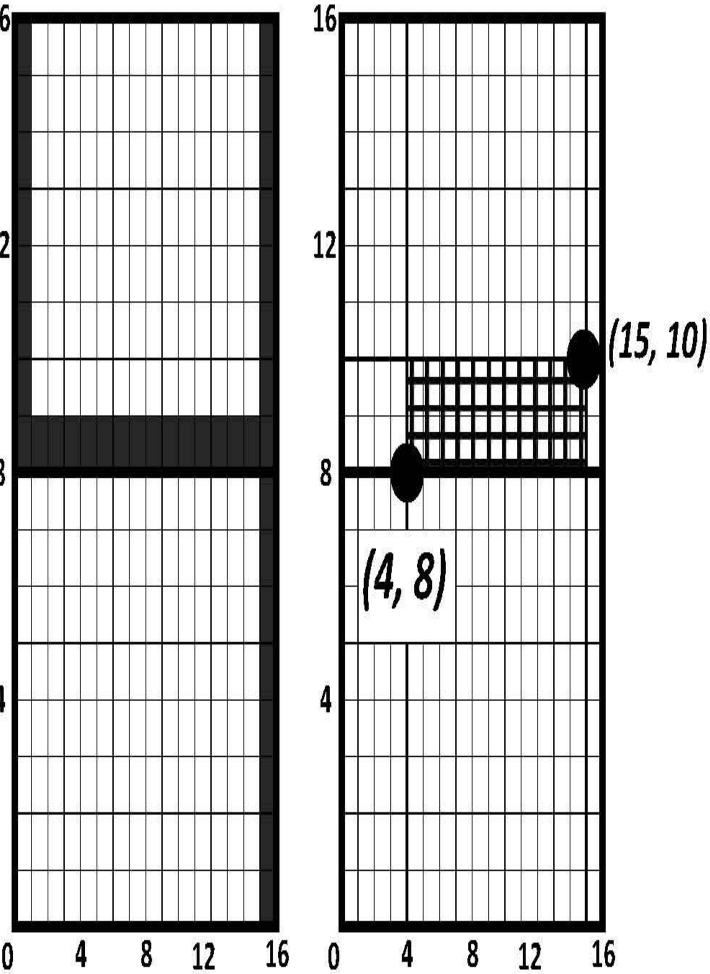
1) **New Tools**

2) **New Theorems and Hypotheses. New realms for Interval Analysis**

3) **New Prerequisites of Applications**

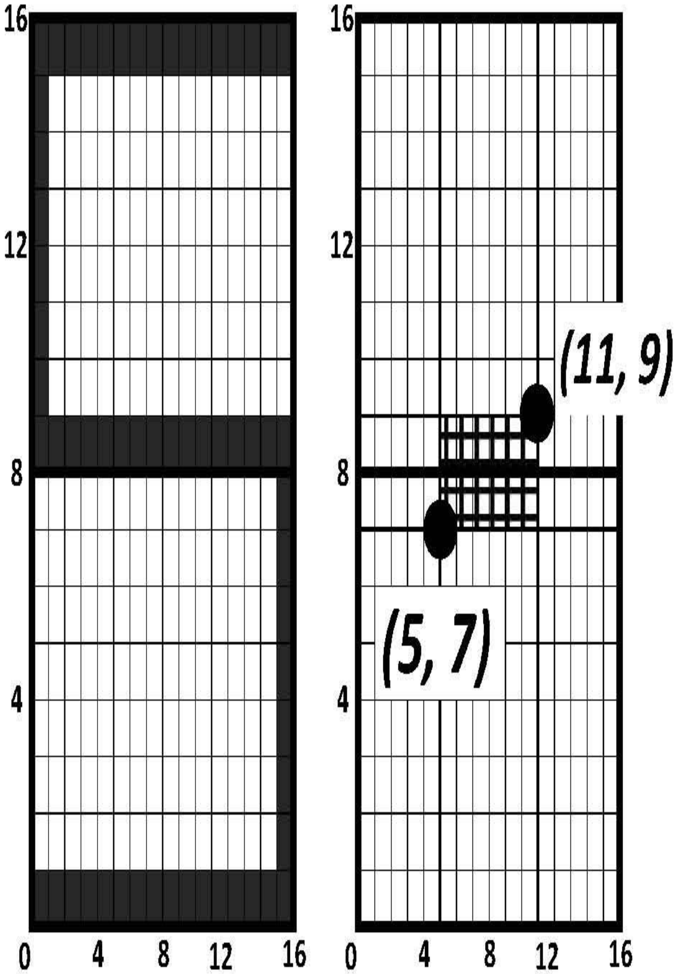
4) **New Fields of Applications**

Sub-Interval Images



(4, 8) (15, 10)

Two bytes

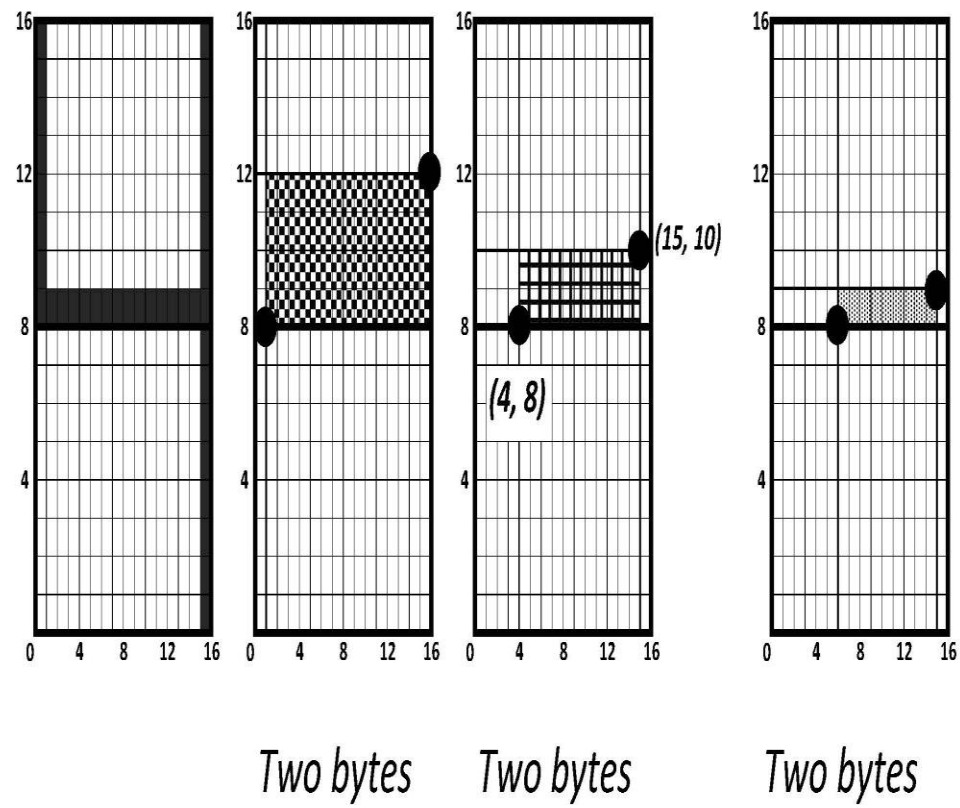


(5, 7) (11, 9)

Two bytes

4

Sub-Interval Images



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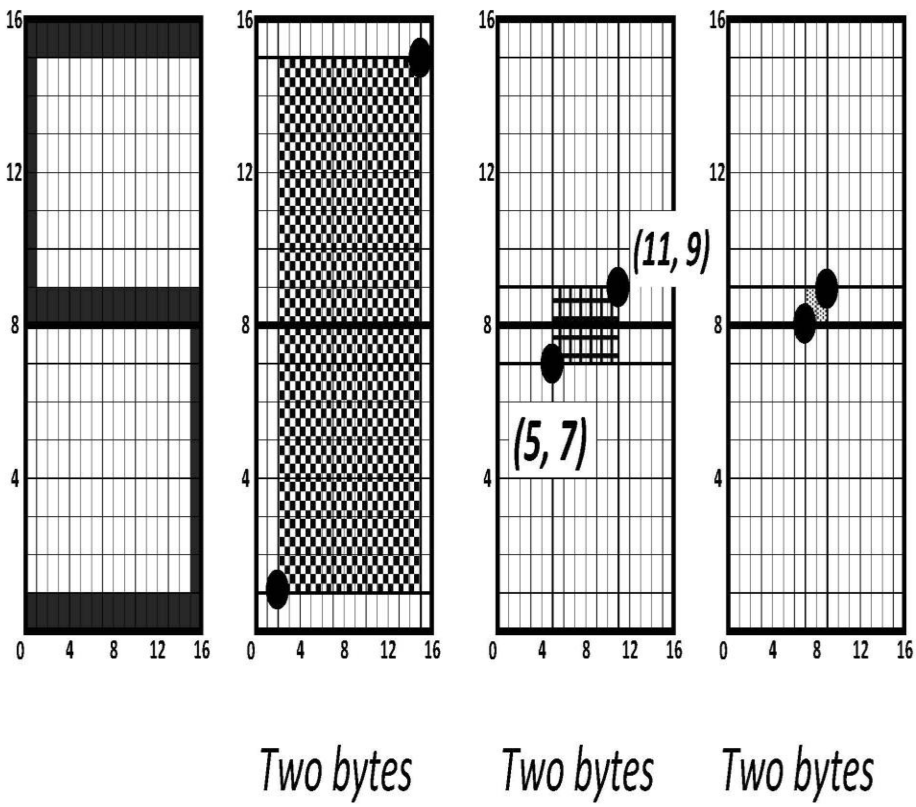
Sub-Interval Images



"Ring of formulas"

$$wid\ M_{1..S} =$$

$$= \sum_{s=1}^{S} w_s wid\ X_s =$$

$$= wid\ X_{1..S} - \sum_{s=1}^{S} w_s \sum_{m=1,..,S|m\neq s} wid\ X_m =$$

$$= wid\ X_{1..S} - \sum_{s=1}^{S} wid\ X_s \sum_{m=1,..,S|m\neq s} w_m =$$

$$= wid\ M_{1..S}$$

What for is It?

1) To treat Inexact Data

2) To treat Incomplete Data

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What for is It?

What for is
the Sub-Interval Analysis?

To treat Incomplete Data

Incomplete Data Analysis

Incomplete Data Analysis
An illustrative example

Consider an interval ***X****=[A, B] =[0, 10]*. Let the only or first measurement gives the weight $w_{First}=0.7$ of the interval $\boldsymbol{X}_{\boldsymbol{First}}=[2, 4]$. Then for interval $\boldsymbol{M}_{\boldsymbol{1..S}}$ of mean value we have

$$\underline{M_{1..S}} \geq$$

$$\geq \underline{X_{1..S}} + wid(\underline{X_{First}} - \underline{X_{1..S}})\ w_{First} =$$

$$= 0 + 2 \times 0.7 = 1.4$$

and

$$\overline{M_{1..S}} \leq$$

$$\leq \overline{X_{1..S}} - wid(\overline{X_{1..S}} - \overline{X_{First}})\, w_{First} =$$

$$= 10 - 6 \times 0.7 = 5.8$$

Incomplete Data Analysis
An illustrative example

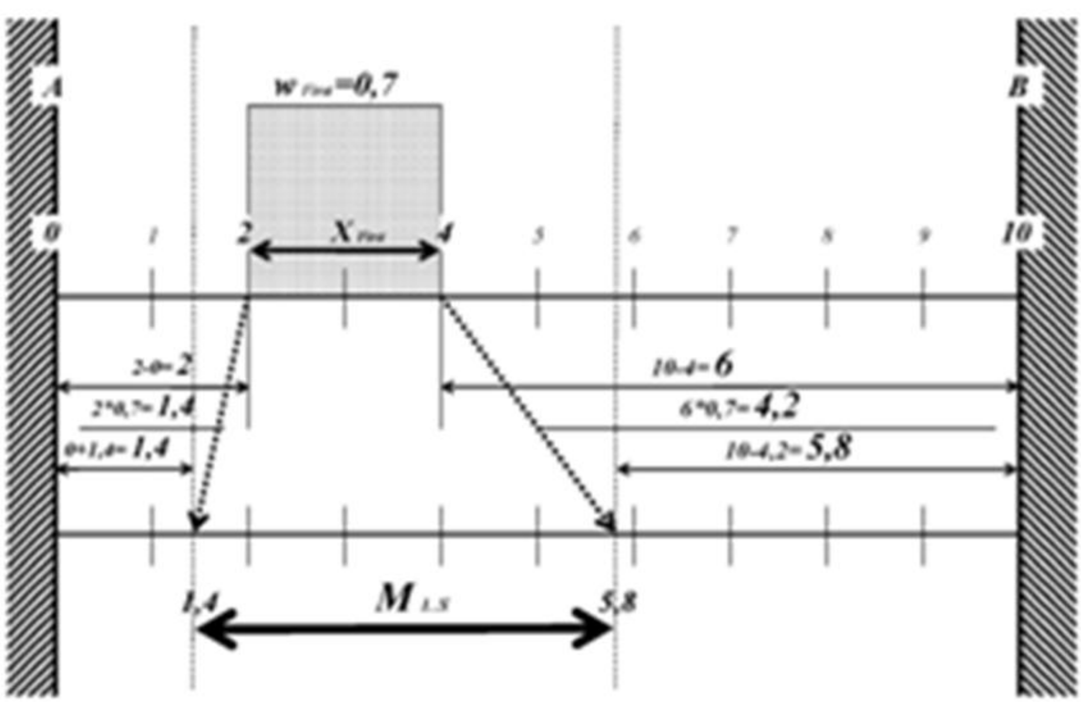
and

$$1.4 \leq$$

$$\leq wid\, M_{1..S} \leq wid\, X_{1..S} -$$

$$- w_{First}(wid\, X_{1..S} - wid\, X_{First}) =$$

$$= 10 - 0.7 \times 8 = 4.4$$



Theorem of Interval Character of Incomplete Knowledge

Theorem. If a quantity $\{w(x_k)\}$ is defined on an interval $\boldsymbol{X}=[A, B]$ and $\{w(x_k)\}$ is exactly known in any "exact" point of $\{x_k\}$ (we denote these "exact" points as $\{x_{Exact\_k}\}$) except two "inexact" points $x_{Inexact1}$ and $x_{Inexact2}$, that is

$$\{w(x_k)\} =$$

$$= \{w(x_{Exact\_k})\} \bigcup w(x_{Inexact1}) \bigcup w(x_{Inexact2})$$

the distance between these two points is non-zero $x_{Inexact2}-x_{Inexact1} \geq 2L > 0$ and the values $w(x_{Inexact1})$ and $w(x_{Inexact2})$ may vary in the non-zero interval Δ such as $Max(w(x_{Inexact1}))-Min(w(x_{Inexact1})) \geq \Delta > 0$ and $Max(w(x_{Inexact2}))-Min(w(x_{Inexact2})) \geq \Delta > 0$,

then any moment of $\{w(x_k)\}$ is known within the accuracy not better than a non-zero interval.

Theorem of Interval Character of Incomplete Knowledge

Proof (For short, only for even n powers of moments. See the complete proof, e.g., in [8]). If the distance between these two "inexact" points is non-zero $|x_{Inexact2}-x_{Inexact1}|\geq 2L>0$, then for any x_0 there is at least one of two "inexact" points, say $x_{Inexact1}$, such as $|x_0-x_{Inexact1}|\geq L$.

Let us denote the exactly known parts of moments $E(X-X_0)^n$ as

$$E(X-X_0)^n{}_{Exact} \equiv$$

$$\equiv \sum_{kExact=1}^{K-2}(x_{Exact\_k}-x_0)^n w(x_{Exact\_k})$$

Theorem
of Interval Character
of Incomplete Knowledge

Let us rewrite the general expression for moments as

$$E(X - X_0)^n =$$

$$= \sum_{k=1}^{K} (x_k - x_0)^n w(x_k) =$$

$$= E(X - X_0)^n{}_{Exact} +$$

$$+ (x_{Inexact\,1} - x_0)^n w(x_{Inexact\,1}) +$$

$$+ (x_{Inexact\,2} - x_0)^n w(x_{Inexact\,2})$$

Theorem of Interval Character of Incomplete Knowledge

If the distance between x_0 and $x_{Inexact1}$ is not less than L or $|x_{Inexact1} - x_0| \geq L$, then we obtain for the even n powers of moments

$$
\begin{aligned}
&Max(E(X-X_0)^n) = E(X-X_0)^n{}_{Exact} + \\
&+ Max[(x_{Inexact\,1} - x_0)^n w(x_{Inexact\,1}) + \\
&+ (x_{Inexact\,2} - x_0)^n w(x_{Inexact\,2})] \geq \\
&\geq E(X-X_0)^n{}_{Exact} + L^n Max(w(x_{Inexact\,1})) + \\
&+ (x_{Inexact\,2} - x_0)^n Max(w(x_{Inexact\,2}))
\end{aligned}
$$

Theorem
of Interval Character
of Incomplete Knowledge

and

$$Max(E(X - X_0)^n) - Min(E(X - X_0)^n) \geq$$
$$\geq L^n Max(w(x_{Inexact\,1})) - L^n Min(w(x_{Inexact\,1})) +$$
$$+ (x_{Inexact\,2} - x_0)^n Max(w(x_{Inexact\,2})) -$$
$$- (x_{Inexact\,2} - x_0)^n Min(w(x_{Inexact\,2})) \geq$$
$$\geq L^n (Max(w(x_{Inexact\,1})) - Min(w(x_{Inexact\,1}))) \geq$$
$$\geq L^n \Delta > 0$$

Taking into account any additional “inexact” point can only increase this uncertainty.

The proof has been done.

Widening of realm of interval analysis to any durable processes

The theorems of interval character of incomplete knowledge essentially widen the realm of interval analysis. The principal aspect of these theorems is that after proving it the interval analysis may include the calculations of both inexact and exact but incomplete information.

Therefore, the interval analysis may be applied to analyze, estimate, plan and correct any durable processes which may be both inexact and exact.

Hypothesis of incompleteness for measurements

Real measurement are often hard to be repeated as more as it is necessary, e.g., in the econometrics. The theorem of interval character of incomplete knowledge states that the results of such measurements possess interval character.

The theorem may be transformed to a hypothesis of incompleteness for measurements interpretation.

Hypothesis of incompleteness for measurements

The hypothesis:

A result of a measurement (or results of measurements) may be interpreted as only a part of a series of N measurements such as:

the number N is:

more, than the real number of measurements, or acceptably great, or reliably great;

the boundaries of series results are:

more wide, than the real boundaries of results, or acceptably wide, or reliably wide.

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