Optimization Problem of Equipment Age Structure in the Model of Russia's Unified Energy System Development

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Integral model of developing system

Glushkov V.M. On a One Class of the Dynamic Macroeconomic Models // Upravlyayuschie sistemy i mashiny. 1977. No 2. P. 3–6 (in Russian).

$$\int_{a(t)}^{t} K(t,s)x(s)ds = y(t), \quad t \in [t_0, T]. \tag{1}$$

$x(s)$ is the number of elemens of the developing system and $t - s$ denotes their age;
$K(t,s)$ is an efficiency coefficient of $x(s)$ at the moment t;
$t - a(t)$ is maximum lifetime of system elements;
$y(t)$ is an integral indicator of the system development level.
If $a(t_0) < t_0$, then it is necessary to define $x(t) \equiv x^0(t),\ t \in [a(t_0), t_0)$.

$$\sum_{i=1}^{n} \int_{a_i(t)}^{a_{i-1}(t)} K_i(t,s)x(s)ds = y(t), \quad t \in [0, T], \tag{2}$$

$$a_i'(t) \geq 0, a_0(t) \equiv t, a_n(t) \equiv 0.$$

Apartsin A.S., Sidler I.V. Using the Nonclassical Volterra Equations of the First Kind to Model the Developing // Automation and Remote Control. 2013. Vol. 74, Issue 6. P. 899–910.

Apartsin A.S., Sidler I.V. Integral Models of Development of Electric Power Systems with Allowance for Ageing of Equipments of Electric Power Plants // Electronic Modeling. 2014. Vol. 36, Issue 4. P. 81–88 (in Russian).

Model of the electric power system development

Scalar case

$$\beta_1(t)\int\limits_{t-T_1(t)}^{t} x(s)ds+\beta_2(t)\int\limits_{t-T_2(t)}^{t-T_1(t)} x(s)ds+\beta_3(t)\int\limits_{t-T_3(t)}^{t-T_2(t)} x(s)ds = y(t),\ t\in[t_0,T],$$

$$x(t)=x^0(t),\ t\in[0,t_0); \tag{3}$$

$$x(t)\geqslant 0,\ t\in[t_0,T].$$

$x(t)$ is the commissioning of electric capacities at moment t;
$\beta_i(t)\in[0,1]$ is the efficiency coefficient of age group i, $\beta_1(t)\geqslant\beta_2(t)\geqslant\beta_3(t)\geqslant 0$;
$T_i(t)$ is the upper age limit of group i, $i=\overline{1,3}$;
$y(t)$ is dynamics of power consumption (electric load) specified by the experts for the future;
$x^0(t)$ is the known dynamics of commissioning the capacities on the prehistory $[0,t_0)$;

$$y(t_0)=\beta_1(t_0)\int\limits_{t_0-T_1(t_0)}^{t_0} x^0(s)ds+\beta_2(t_0)\int\limits_{t_0-T_2(t_0)}^{t_0-T_1(t_0)} x^0(s)ds+\beta_3(t_0)\int\limits_{t_0-T_3(t_0)}^{t_0-T_2(t_0)} x^0(s)ds.$$

Take the cost functional as the objective functional:

$$I\left(x(t), T_3(t)\right) \equiv I_1 + I_2, \tag{4}$$

where

$$I_1 = \int\limits_{t_0}^{T} a^{t-t_0} \left\{ \beta_1(t) \int\limits_{t-T_1(t)}^{t} u_1(t-s)u_2(s)x(s)ds + \right.$$

$$\left. +\beta_2(t) \int\limits_{t-T_2(t)}^{t-T_1(t)} u_1(t-s)u_2(s)x(s)ds + \beta_3(t) \int\limits_{t-T_3(t)}^{t-T_2(t)} u_1(t-s)u_2(s)x(s)ds \right\} dt$$

are total operating costs for the forecast period;

$$I_2 = \int\limits_{t_0}^{T} a^{t-t_0} k(t)x(t)dt$$

are total costs for the commissioning of new generating capacities.

In (4)–(4) the following functions are known:

$u_1(t-s)$ is coefficient of increase in the costs of operating the capacities at moment t that are commissioned at moment s;
$u_2(t)$ are specific costs of operating the capacity put into service at moment t;
$k(t)$ are costs of putting into service a unit of capacity at moment t;
a^{t-t_0} is coefficient of discounted costs, $0 < a < 1$.

The control parameter $T_3(t)$ belongs to an fiasible set

$$U = \left\{ T_3(t) : \ \underline{T_3} \leqslant T_3(t) \leqslant \overline{T_3}, \ T_3'(t) \leqslant 1, \ t \in [t_0, T] \right\}. \tag{5}$$

The optimal control problem consists in determination of

$$T_3^*(t) = arg \min_{T_3(t) \in U} I(x(t), T_3(t)), \tag{6}$$

under conditions (3)-(5).

To solve the optimal control problem, we use a heuristic algorithm based on the discretization of all the elements on a grid with step $h = 1$ (year) and replacement of the feasible set U by the set U_h of piecewise linear functions:

$$T_3(t) = \begin{cases} m, \ t \in (t_0, T], & m \leqslant T_3(t_0), \\ t - t_0 + T_3(t_0), \ t \in [t_0, t_0 - T_3(t_0) + m), & m > T_3(t_0), \\ m, \ t \in [t_0 - T_3(t_0) + m, T], & \end{cases} \qquad (7)$$

where m is desired integer constant lifetime.

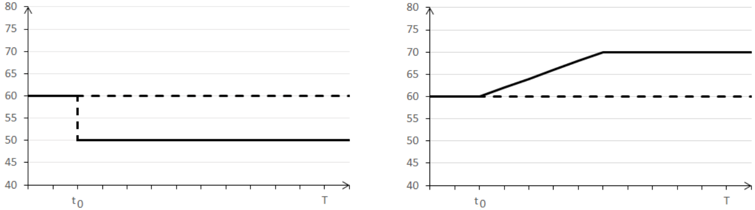

Fig. 1.

We set the forecast values of economic indices based on expert estimates. For the given $T_3(t) \in U_h$, from the balance Eq. (3) we can determine the corresponding $x(t)$ and calculate the functional $I(m) \equiv I(x(t), T_3(t))$.

Model of the electric power system development

Vector case: 1 – TPP, 2 – NPP, 3 – HPP

$$\sum_{i=1}^{3}\left(\beta_{i1}\int_{t-T_{i1}(t)}^{t} x_i(s)ds+\beta_{i2}\int_{t-T_{i2}(t)}^{t-T_{i1}(t)} x_i(s)ds+\beta_{i3}\int_{t-T_{i3}(t)}^{t-T_{i2}(t)} x_i(s)ds\right)=y(t),\quad t\in[t_0,T], \tag{8}$$

$$\int_{t-T_{13}(t)}^{t} x_1(s)ds=\alpha(t)\left(\int_{t-T_{13}(t)}^{t} x_1(s)ds+\int_{t-T_{23}(t)}^{t} x_2(s)ds+\int_{t-T_{33}(t)}^{t} x_3(s)ds\right), \tag{9}$$

$$\int_{t-T_{33}(t)}^{t} x_3(s)ds=\gamma(t)\left(\int_{t-T_{13}(t)}^{t} x_1(s)ds+\int_{t-T_{23}(t)}^{t} x_2(s)ds+\int_{t-T_{33}(t)}^{t} x_3(s)ds\right), \tag{10}$$

$$x(t)=x^0(t),\ t\in[0,t_0),$$

$$x(t)\geqslant 0,\ t\in[t_0,T].$$

$x(t)\equiv(x_1(t),x_2(t),x_3(t))$ is the commissioning of electric capacities (by types of power plants);
$y(t)$ is total available capacity of the EPS specified by the experts for the future;
$T_{ij}(t)$ is the upper age limit of group j for a power plant of type i;
$x^0(t)\equiv(x_1^0(t),x_2^0(t),x_3^0(t))$ is the known dynamics of commissioning the capacities on the prehistory $[0,t_0)$;
$\alpha(t)$ is change in TPP's part of capacity in the total composition of generating equipment;
$\gamma(t)$ is change in HPP's part of capacity in the total composition of generating equipment.

Forecasting problem

Base variant:

$T_{i1} = 30,\ T_{i2} = 50,\ T_{13} = T_{23} = 60,\ T_{33} = 101$

$(T_{13}(t_0) = 60,\ T_{23}(t_0) = 47,\ T_{33}(t_0) = 66),$

$\beta_{i1} = 1,\ \beta_{i2} = 0,97,\ \beta_{i3} = 0,9,\ i = \overline{1,3}, \quad \alpha(t) = 0,69,\ \gamma(t) = 0,19 \quad t \in [2016, 2050]$

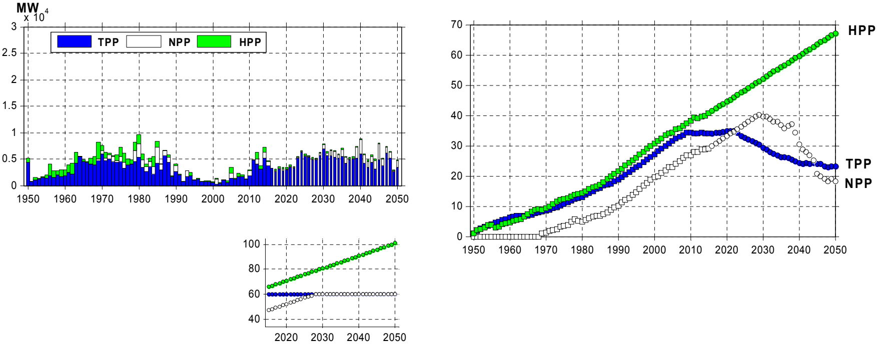

Fig. 2. The dynamics of commissioning capacities and the average age of generating capacities, the growth rate of $y(t)$ is 0,5%.

The objective functional:

$$I\left(x\left(t\right),\,T_3\left(t\right)\right)\equiv I_1+I_2, \tag{11}$$

where

$$I_1=\sum_{j=1}^{3}\int\limits_{t_0}^{T}a^{t-t_0}\left\{\sum_{i=1}^{3}\beta_{ji}(t)\int\limits_{t-T_{ji}(t)}^{t-T_{j,i-1}(t)}u_1^j(t-s)u_2^j(s)x_j(s)ds\right\}dt,\ \ T_{j0}=0, \tag{12}$$

are total operating costs for the forecast period;

$$I_2=\sum_{j=1}^{3}\int\limits_{t_0}^{T}a^{t-t_0}k_j(t)x_j(t)dt \tag{13}$$

are total costs for the commissioning of new generating capacities.

Optimization of equipment lifetimes for TPPs and NPPs. The HPP equipment is not decommissioned for the forecast period.

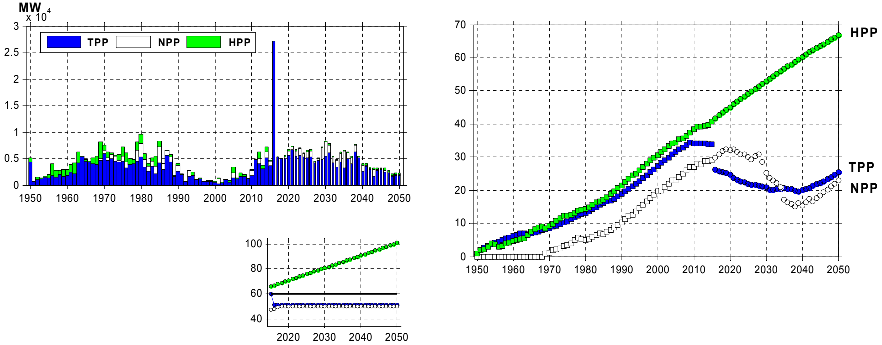

Fig. 3. $I(T_3^*) = I(51, 50, 101) = 96,73\% \cdot I(60, 60, 101)$.
The growth rate of $y(t)$ is 0,5%, $u_1^1 = 3\%$, $u_1^1 = 10\%$, $u_1^1 = 3\%$.

Optimization problem with constrains on $x(t)$

Remark. We considered the problem without upper limits on commissioning of the capacities. If we introduce additional constraints on the phase variable

$$x(t) \leqslant \bar{x}(t), \; t \in [t_0, T], \tag{14}$$

then the optimal strategy will have a smoother transition to a constant lifetime, and in general will give a smaller benefit relative to the base case.

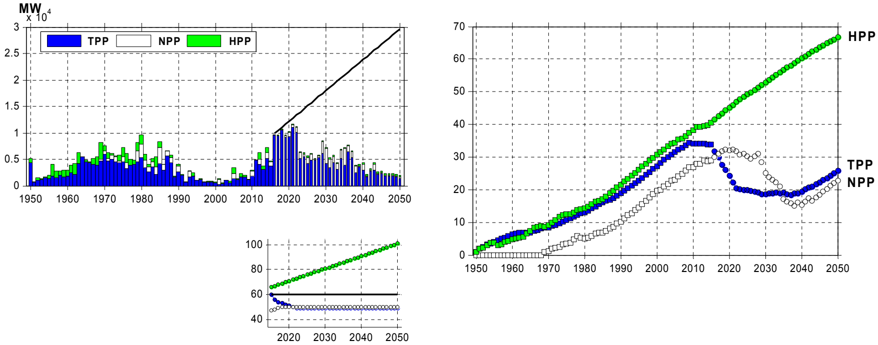

Fig. 4. $I(T_3^*) = I(49^*, 50, 101) = 96,73\% \cdot I(60, 60, 101)$.

We consider two different scenarios of energy consumption as an energy strategy:
– the optimistic scenario, providing a high level of electricity consumption (the growth rate of $y(t)$ is 2% per year);
– the realistic scenario, with a low level of energy consumption (the growth rate is 0,5%).
The second aspect of our research is the influence of changes in the specific operation cost on the optimal solution under increasing the equipment lifetime. In particular, the influence of the coefficient u_1^1 was investigated: u_1^1 increases the cost of operating TPP capacities from 1% to 3% per year after 45 years of service (u_1^2 and u_1^3 are fixed, the costs increase of 10% and 3%, respectively).

Table: Optimal lifetime for TPP, years

Growth rate of $y(t)$	u_1^1			
	1%	1,5%	2%	3%
0,5%	4,55% **95**	2,31% **95**	2,26% **53**	3,27% **51**
2%	3,37% **95**	1,71% **95**	1,68% **53**	2,45% **50**

Optimal lifetime of nuclear power plants plans a transition from 47 to 50 years.

Study the influence of economic indicies on the solution of the optimal control problem

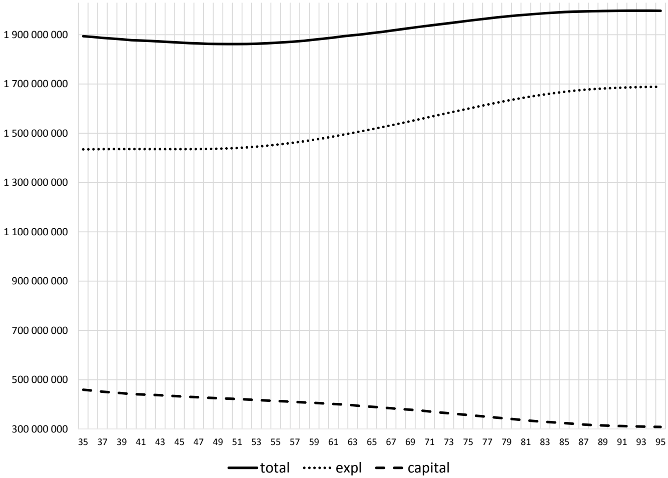

Fig. 5. Dependencies of costs on maximum lifetime for TPP (lifetimes for NPP and HPP are base). The growth rate of $y(t)$ is 2%, u_1^1 is 3%.

Study the influence of economic indicies on the solution of the optimal control problem

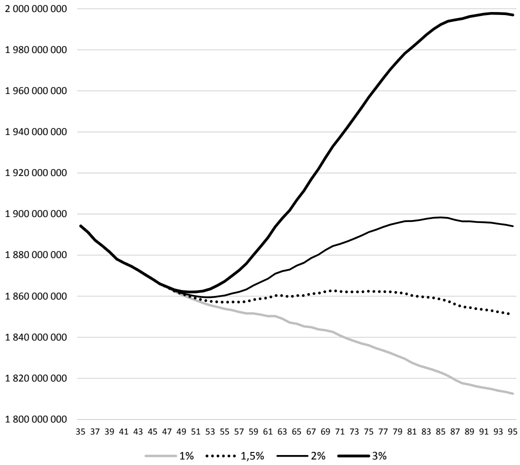

Fig. 6. Dependencies of total costs on maximum lifetime for TPP for different options u_1^1.

Thank you for attentions!