On Hybrid Method for Medium-Term Multi-Product Continuous Plant Scheduling

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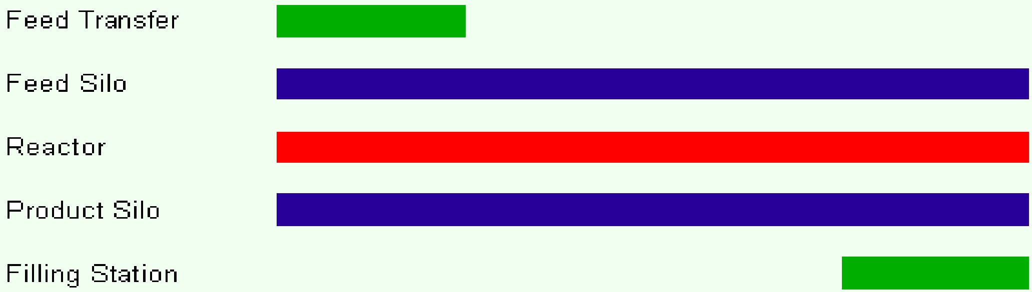
Plant structure

Feed Transfer -> Feed Silo -> Extruder -> Product Silo -> Filling Unit

Example of a single production stage



Special conditions:

- Sequence dependent setup times for extrusions
- Weekend ban filling of some products

Solution Approach

Decomposition Scheme:
The whole scheduling period is decomposed to half-day horizons.

Upper-level optimization model: choose the products to be planned in each horizon.

Lower-level optimization model: Construct the schedule in the horizon.

Shaik, M.A., Floudas, C.A., Kallrath, J., Pitz, H.-J., Production scheduling of a large-scale industrial continuous plant: short-term and medium-term scheduling. Computers & Chemical Engineering (2008) In press. Scheduling.

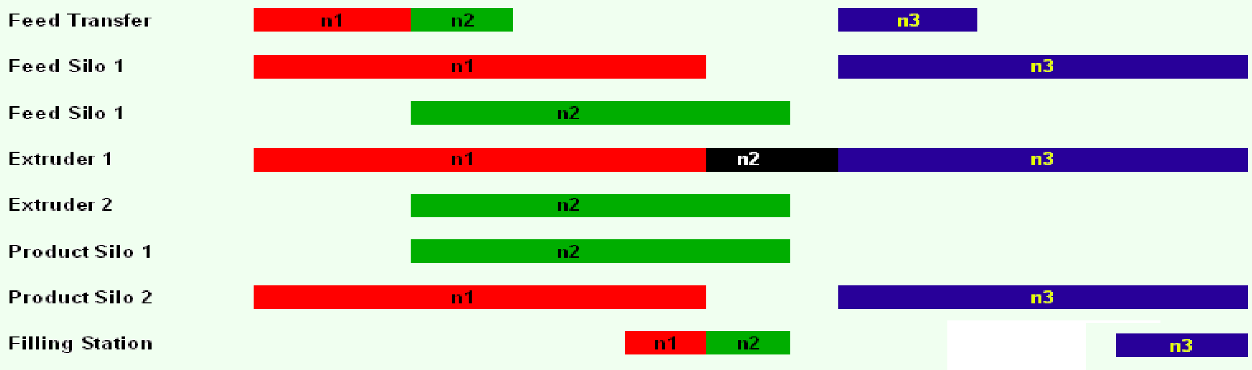
Lower-level optimization model

Binary variables:

$\delta_{i,n}$ equals 1 if task i is allocated at event point n.

Positive variables:

$b_{i,n}$ is the amount produced by task i at event point n.
$t^s_{i,n}$ is the start task i at event point n.
$t^s_{i,n}$ is the finish task i at event point n.



16 tasks (ft_red, ft_green, ft_blue, fs_red, fs_green, fs_blue, e_red, e_green, e_blue, ps_red, ps_green, ps_blue, pf_red, pf_green, pf_blue, ch_black),
3 event pounts
Number of binary variables is $16 \times 3 = 48$.

Lower-level optimization model: System of constraints

- **Allocation of tasks in event points**

  One event point contains not more that one task;

  If material is transferred from one task to another, then both tasks are located in the same event point;

- **Material balance constraints**

  Produced amount must fit minimal and maximal capacities of a unit;

  If material is transferred from one task to another, then produced amount is equal to consumed amount;

- **Timing constraints**

  Duration of a task depends on its amount;

  Tasks on the same unit do not overlap;

  If material is transferred from one task to another, the tasks must be synchrinized in time;

  Sequnce dependent changeovers, week-end ban, etc.

Hybrid Approach: I Optimizing the extrusion tasks

$\boldsymbol{S = \{1, 2, ..., n\}}$ is the set of products;
$\boldsymbol{U = \{1, 2, ..., m\}}$ is the set of units;
$\boldsymbol{I \ = \{1, 2, ..., l\}}$ is the set of tasks;

$\boldsymbol{D_s}$ is the demand on product $\boldsymbol{s}$;

For each task $\boldsymbol{i \in I}$ it is given:
$\boldsymbol{u_i}$ is the suitable unit (one only)
$\boldsymbol{s_i}$ is the output product (one only)
$\boldsymbol{r_i}$ is the production rate.
$\boldsymbol{T_i^{min}}$ is the minimal possible duration of task $\boldsymbol{i}$

$\boldsymbol{s_{ij}}$ is the setup time if $\boldsymbol{i}$ and $\boldsymbol{j}$ are performed on the same unit

Objective: minimize Makespan

Genetic Algorithm

Representation of solutions

$\boldsymbol{\Pi = (i_1, i_2, ... i_l)}$ tasks permutation;
$\boldsymbol{L = (L_1, L_2, ..., L_m)}$, where $\boldsymbol{L_u}$ is the maximal allowed number of tasks to be placed on unit u;

Example:
Tasks 1,2,3,4 can be performed on unit 1,
Tasks 5,6,7,8,9 can be performed on unit 2.

Some possible solution

$$\boldsymbol{\Pi = (4, 7, 5, 2, 3, 9, 1, 8, 6)}$$
$$\boldsymbol{L = (2, 3)}$$

corresponds to the following assignment:

Unit 1:	4,2
Unit 2:	7,5,9

LP formulation to complete the schedule

Solution $(\Pi, L,)$ defines the set of tasks to be performed, and the sequence of tasks. The changeover times can be calculated straightforwardly. To complete the schedule it is enough to determine the **durations** of the tasks.

Let $I(\Pi, L, s)$ be the set of tasks producing product s,
$Ch(\Pi, L, u)$ be the sum of changeover time on unit u.

Recall that r_i is a production rate of task i.
D_s is a demand on product s.

Positive variables: $\tau_i \geq 0$ is the duration of task i.

LP Model

Objective: minimize Makespan

$$\min C_{\max}$$

Subject to:

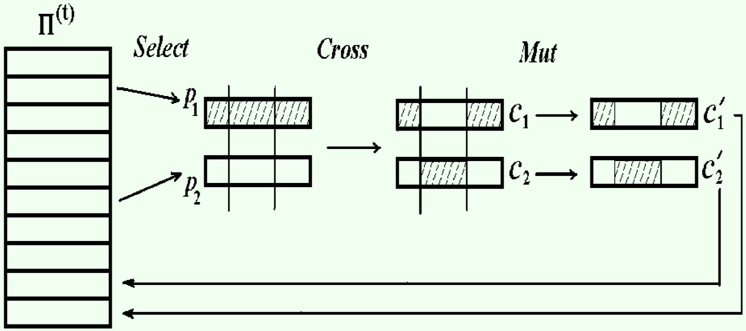
1. Demand satisfaction constraint:

$$\sum_{i \in I(\Pi, L, s)} r_i \tau_i \geq D_s.$$

2. Estimation of Makespan:

$$\sum_{i \in T} \tau_i \leq C_{\max} - Ch(\Pi, L, u).$$

Scheme of Genetic Algorithm



1. Generate initial population $\boldsymbol{\Pi}^{(0)}$.
2. For $\boldsymbol{t := 1}$ to $\boldsymbol{t_{max}}$ do

    2.1 $\boldsymbol{\Pi}^{(t)} = \boldsymbol{\Pi}^{(t-1)}$.

    2.2 Apply **selection** operator to choose $\boldsymbol{p_1, p_2}$ from $\boldsymbol{\Pi}^{(t)}$.

    2.3 Apply **crossover** operator to $\boldsymbol{p_1}$ и $\boldsymbol{p_2}$ to generate new solutions $\boldsymbol{c_1}$ and $\boldsymbol{c_2}$.

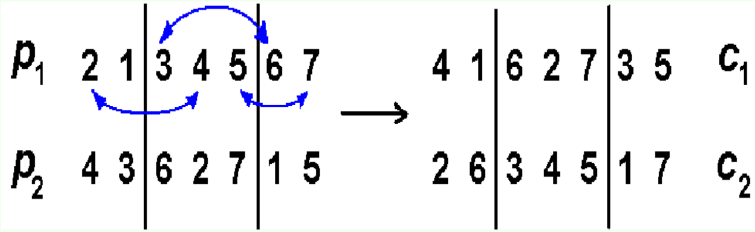
    2.4 Apply **mutation** operator: $\boldsymbol{c_1' = \mathcal{M}(c_1)}$ и $\boldsymbol{c_2' = \mathcal{M}(c_2)}$.

    2.5 Remove two worst solutions $\boldsymbol{q_1, q_2}$ from $\boldsymbol{\Pi}^{(t)}$ and add $\boldsymbol{c_1'}$ and $\boldsymbol{c_2'}$.

GA operators

Tournament selection: Choose randomly $\boldsymbol{s}$ solutions and return the best among them.

Partially mapped crossover (PMX):



Mutation: Choose randomly $\boldsymbol{k}$ and $\boldsymbol{l}$, and exchange $\boldsymbol{j_k}$ and $\boldsymbol{j_l}$.

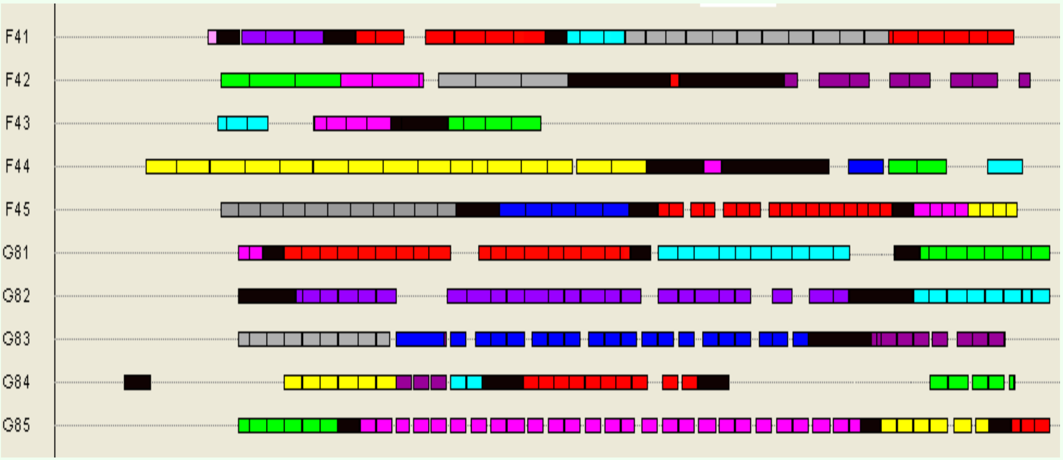
Hybrid Approach: II Solving the basic problem

Decomposition Scheme:
Weekday horizons (5 days)
Weekend horizons (2 days)

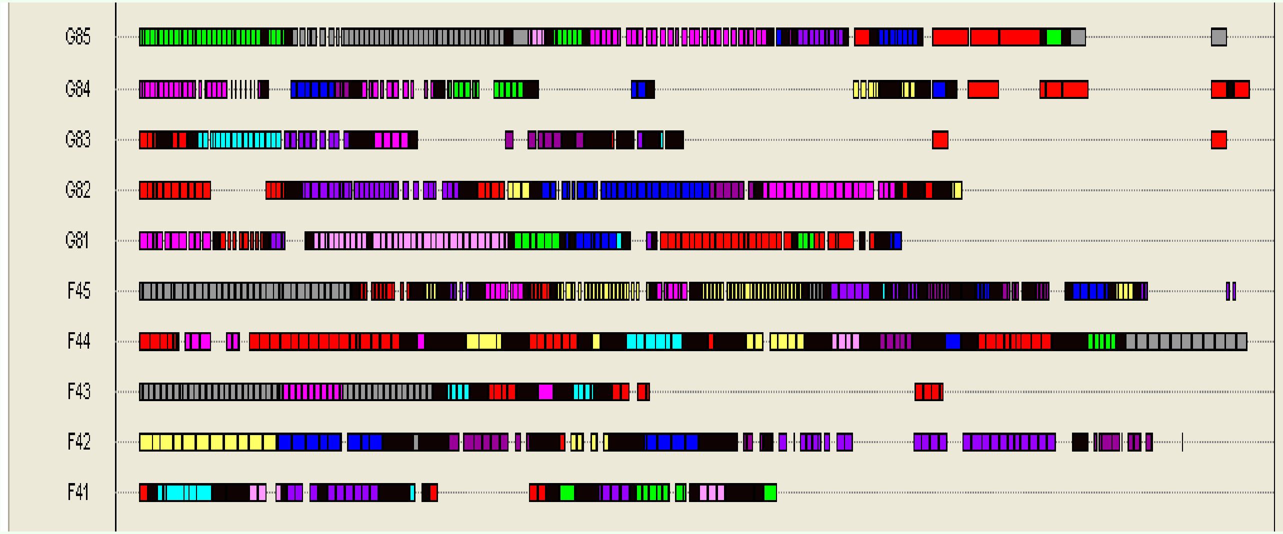
Solving the short-term problem:

1. Start from empty schedule (fix all binary variables to zero).
2. Run the GA to find a schedule for extruders.
3. Take the earliest extrusion task from the GA schedule.
4. Unfix the binary variables corresponding to the selected extrusion task and all the auxilary tasks related to it.
5. Solve the subproblem and fix the binary variables to their values.

Genetic Solution and the Actual Schedule



Whole schedule for extruders



Extrusion tasks in the solution: 1057
All tasks in the solution: 4697
Event points used: 211

Experimental Results

Articles: 120
Extruders: 10
Extrusion tasks: 172

	Pure Decomposition	Decomposition + Greedy	Hybrid
Underproduction, %	0.96	0.02	0.07
Number of changeovers	324	194	115
Total changeovers duration	930	851	490
Solving time	16h	12h	6h