The exact definition of the backstress in the compression problem of a flat or cylindrical poroelastic layer

Sofia Sizova, S.V. Golovin

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Main equations of poroelasticity

1. Governing Equations

$$\operatorname{div} \boldsymbol{\sigma} = 0, \quad \boldsymbol{\sigma} = \lambda \operatorname{div} \mathbf{u}\, \mathbf{I} + 2\mu\, \mathcal{E}(\mathbf{u}) - \alpha p\, \mathbf{I} \tag{1}$$

2. Relation between pore pressure, deformation of the skeleton and fluid compressibility

$$p = M(-\alpha\epsilon + \rho_f \phi) \tag{2}$$

3. Filtration of fluid

$$\frac{\partial p}{\partial t} = M \operatorname{div} \left(\frac{k}{\eta} \nabla p - \alpha \frac{\partial \mathbf{u}}{\partial t} \right) \tag{3}$$

Here $\boldsymbol{\sigma}$ — Cauchy stress tensor, $\mathbf{u}$ — the displacement vector of the skeleton, $\mathcal{E}(\mathbf{u})$ — stress-strain tensor, $\epsilon = \operatorname{tr} \mathcal{E}$ — volume deformation, $\alpha = 1 - K/K_s$ — Biot coefficient, λ и μ — elasticity modulus, M — Biot modulus, ϕ — porosity, k — permeability, η — fluid viscosity.

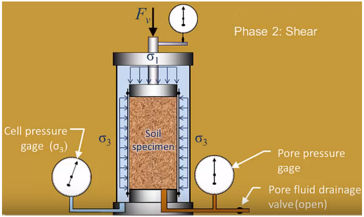
Drained and undrained experiments

Drained experiment

$dp = 0$ or $t \to \infty$

at constant porous pressure

$$d\sigma = K d\epsilon,$$
$$d(\rho_f \phi) = \alpha d\epsilon.$$



Undrained experiment

$d(\rho_f \phi) = 0$ or $t = 0^+$

no movement of the fluid

$$d\sigma = K_u d\epsilon,$$
$$dp = \alpha M d\epsilon.$$

$K_u = K + \alpha^2 M$ — undrained bulk modulus: $K_u > K$.

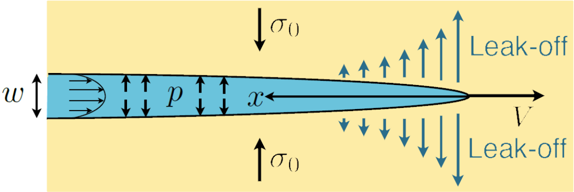
$$dp = -B d\sigma$$

Here $B = \dfrac{\alpha M}{K_u}$ is Skempton coefficient, $0 < B < 1$.

Conclusion: porous fluid increases effective elasticity of the material!

The problem of the stimulation of the hydraulic fracturing.

Hydraulic fracture opens under the pressure of the pumped fluid.



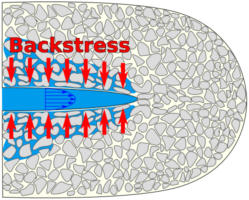
One of problems is finding the opening of the fracture w as a function of fluid pressure p.

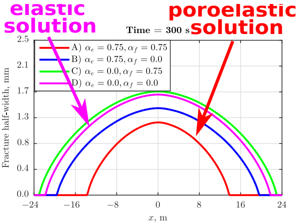
$$w = F[p; \lambda, \mathbf{u}, \alpha, M, \sigma_0, \ldots]$$

at given poroelastic reservoir characteristics and compressive rock pressure σ_0.

Influence of the backstress on the opening of the fracture

Backstress is the additional compressive stress, which appear due to the filtration of the fluid into the reservoir





Backstress in the fracture (left) and opening of the fracture at different α (right)

Influence of the poroelastic effects decreases the effective length of the fracture:

Generation of the backstress $\Rightarrow$ Increase of pressure in the fracture $\Rightarrow$

Increase of the leakoff $\Rightarrow$ Decrease of fracture length

Formal definition of the backstress

We will distinguish explicitly the contribution of the pore pressure to stresses in the medium:

$$\mathbf{u} = \mathbf{u}^r + \mathbf{u}^p, \quad \boldsymbol{\sigma} = \boldsymbol{\sigma}^p + \boldsymbol{\sigma}^r - \alpha p I.$$

Equilibrium equation are divided into 2 subproblems:

$$\operatorname{div} \boldsymbol{\sigma}^p = \alpha \nabla p, \qquad \Gamma_f : \mathbf{u} = 0$$

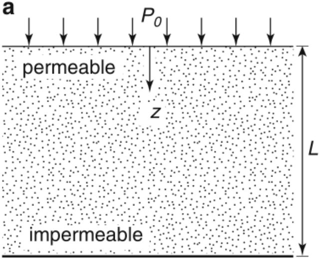
$$\operatorname{div} \boldsymbol{\sigma}^r = 0, \qquad \Gamma_f : \boldsymbol{\sigma}^r \langle \mathbf{n} \rangle = (\sigma_0 - p)\mathbf{n} + \underline{\alpha p\, \mathbf{n} - \boldsymbol{\sigma}^p \langle \mathbf{n} \rangle}.$$

Backstress $\mathbf{b} = \left(\alpha p\, \mathbf{n} - \boldsymbol{\sigma}^p \langle \mathbf{n} \rangle \right)\Big|_{\Gamma_f}.$

Taking into account the backstress allows to solve the problem of crack opening in the framework of the usual elasticity model for $\boldsymbol{\sigma}^r$ with additional stress $\mathbf{b}$ over the fracture's wall.

Defining of the Backstress in Exact Solutions

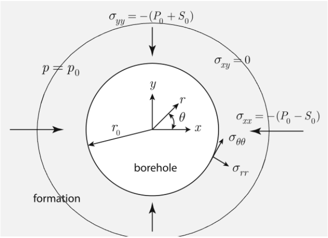
Compression of the flat poroelastic layer under the fluid pressure



$$z = L: \quad w = 0, \frac{\partial p}{\partial z} = 0,$$
$$z = 0: \quad \boldsymbol{\sigma}\langle \mathbf{n} \rangle = -P_0 \mathbf{n},$$
$$t = 0: \quad w = 0, p = 0.$$

$$0 < L < \infty$$

Compression of the cylindrical poroelastic layer



$$r = R_{in}:$$
$$\sigma_{rr} = -P_0 + S_0 \cos 2\theta,$$
$$\sigma_{r\theta} = -P_0 + S_0 \sin 2\theta, \; p = -P_0$$
$$r = R_{out}: w = 0.$$

Compression of the flat layer

Equilibrium conditions for tensor $\boldsymbol{\sigma}^p$:

$$(\lambda + 2\mu)\frac{\partial^2 w^p}{\partial z^2} = \alpha\frac{\partial p}{\partial z}, \quad w^p|_{z=0} = w^p|_{z=L} = 0.$$

Consequently,

$$\mathbf{n} \cdot \boldsymbol{\sigma}^p\langle\mathbf{n}\rangle|_{z=L} = (\lambda + 2\mu)\frac{\partial w^p}{\partial z}|_{z=L} = \alpha p_0(t) - \frac{\alpha}{L}\int_0^L p(t,s)ds.$$

According to the definition we will find backstress in the form:

$$\mathbf{b} = (\alpha p\mathbf{n} - \mathbf{n} \cdot \boldsymbol{\sigma}^p\langle\mathbf{n}\rangle)|_{z=L} = \frac{\alpha}{L}\int_0^L p(t,s)ds\,\mathbf{n}.$$

Backstress is proportional to the average porous pressure over the layer!

Evolution of porous pressure

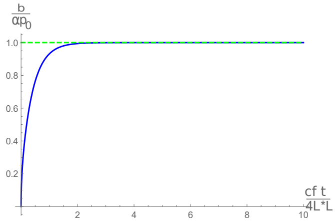
Exact solution for pressure:

$$p = p_0 \left(1 - \sum_{n=0}^{\infty} \frac{4}{\pi(2n+1)} \sin\left(\frac{\pi(2n+1)}{2L} z\right) \exp\left(-\frac{\pi^2(2n+1)^2}{4L^2} c_f t\right)\right)$$

Here

$$c_f = \frac{k}{\eta} M \frac{K + \frac{4}{3}\mu}{K_u + \frac{4}{3}\mu}$$ — coefficient of the filtration of the porous media

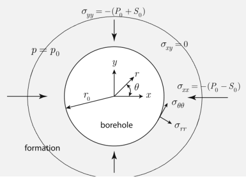
The position of filtration front: $z_{\text{filtr}} \sim \sqrt{c_f t}$.
The leakoff velocity: $v_l \sim 1/\sqrt{c_f t}$



Cylindrical layer: the axisymmetric case

We will suppose $S_0 = 0$.
Radial displacement

$$u_r = \frac{\alpha(1-2\nu)}{2G(1-\nu)} \frac{1}{r} \int_{R_{in}}^{r} (sp(s,t)ds) + A_1(t)r + \frac{A_2(t)}{r}.$$



Coefficients A_1 and A_2 are determined by the boundary conditions.
The value of the backstress is the same on both the external and the internal cylinder and is proportional to the average pressure.

$$\mathbf{b} = \frac{2\alpha \int_{R_{in}}^{R_{out}} s\, p(s,t)ds}{R_{out}^2 - R_{in}^2} \mathbf{e}_r$$

Backstress for the symmetrical compression is equal to zero for the cases of an infinite flat and cylindrical layers!

Cylindrical layer: non-axisymmetric case

We will consider that $p_0 = 0,\ S_0 \neq 0$.

$$\mathbf{b}_r = 2\alpha(-1+2\nu)\Big(\cos 2\theta \frac{m\lambda^2 + m + 2\lambda^4(6-4\nu)}{m^2\lambda^6 + 3\lambda^2 - m^2}\int_\lambda^1 \frac{P}{\tilde{s}}d\tilde{s} + \frac{3+3\lambda^2 m}{\lambda^6 m^2 - 3\lambda^4 + 3\lambda^2 - m^2}\int_\lambda^1 P\tilde{s}^3 d\tilde{s}\Big), \tag{4}$$

$$\mathbf{b}_\theta = 2\alpha(-1+2\nu)\sin 2\theta\Big(\frac{4\nu\lambda^4 - \lambda^2 m - m}{3\lambda^2 - 3\lambda^4 + m^2\lambda^6 - m^2}\int_\lambda^1 \frac{P}{\tilde{s}}d\tilde{s} - \frac{3+3\lambda^2 m}{\lambda^6 m^2 - 3\lambda^4 + 3\lambda^2 - m^2}\int_\lambda^1 P\tilde{s}^3 d\tilde{s}\Big). \tag{5}$$

Here $\lambda = \frac{R_{in}}{R_{out}},\ m = 3 - 4\nu$.

Cylindrical layer: the non-axisymmetric case

For the infinite layer $R_{\rm out} = \infty$,

$$\mathbf{b}_r = -\frac{2\alpha(2\nu-1)\cos 2\theta}{3-4\nu}\int_{R_{in}}^{\infty}\frac{P}{s}ds,$$
$$\mathbf{b}_\theta = \frac{2\alpha(2\nu-1)\sin 2\theta}{3-4\nu}\int_{R_{in}}^{\infty}\frac{P}{s}ds$$

Backstresses

- exist in the infinite layer for the inhomogeneous porous pressure;
- have both normal and tangential components;
- determined by the Poisson coefficient and does not depend on the Young's modulus

Conclusions

- The formal definition of the backstresses, which appear due to the action of the porous pressure over the boundaries of the domain, is given;
- The exact formulas for the backstress in the problems of the compression of flat and cylindrical layers are found;
- The analysis of the observed formulas revealed
  - For the homogeneous distribution of the pressure, the backstress is proportional to the average stress at a layer and is equal to zero for the infinite layer;
  - Under the inhomogeneous porous pressure, the backstress has both normal and tangential components, depends on the Poisson coefficient and is proportional to the integral from the porous pressure.
- The leakoff velocity and the backstress are strongly influenced by the poroelastic filtration coefficient.