Modelling slurry flow in a hydraulic fracture: from mathematical model to deployable software

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Hydraulic fracturing is a complex process that incorporates several physical subprocesses with phenomenas that have different spacial and time scales. If one wants to model the whole process different mathematical models for different subprocesses have to be taken into account. Here the main focus will be on the model of fluid propagation in the fracture, but also some light will be shed on how to incorporate it into the whole hydraulic fracturing simulation pipeline.

The fluid flow in a Hele — Shaw cell is described using a single–velocity model which results in the following two equations [1]:

$$\frac{\partial w}{\partial t} + \nabla(w\mathbf{u}) = -q_l, \quad \mathbf{u} = \frac{w^2}{12\mu(c)}\nabla p; \tag{1}$$

$$\frac{\partial (cw)}{\partial t} + \nabla(cw\mathbf{u}) = 0. \tag{2}$$

Here w is the fracture width, $\mathbf{u}$ the velocity field, q_l the fluid leak-off through the cell walls, p the pressure, $\mu(c)$ the effective viscosity and c the proppant concentration.

Equation (1) reduces to a simple elliptic equation for the pressure p. In the context of the whole hydro fracturing simulation w is assumed to be known at any given time. Equation (2) has a different type and is a transport equation for the concentration of proppant in the moving slurry. The difficulty here is to solve these equations as a coupled model. For (1) a straightforward finite–volume method is used. Using a five–point approximation stencil this results in a SLAE that has to be solved either directly or iteratively. Solving this system gives the pressure p from which the velocities $\mathbf{u}$ can be derived. Considering velocities are now known (2) is solved numerically using a second–order direct [2] scheme.

This algorithm has been implemented using the C++ language and packed into a dynamic library for external use. When preparing this method for integrating into the hydrofracturing simulation pipeline one faces numerous technical and several algorithmic issues: a changing fracture geometry has to be considered, several fluids and proppants are moving simultaneously, the leak-off is changing rapidly and the fact that this algorithm is decoupled from the fracture propagation simulation restricts non–linear iteration usages.

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References

[1] Economides M. J., Nolte K. G. *Reservoir stimulation. 3rd edition.* Chichester: John Wiley & Sons Ltd, 2000.

[2] Leveque R. J. *High-resolution conservative algorithms for advection in incompressible flow.* SIAM J. Numer. Anal. 1996. V. 332. No. 2 P. 627–665.