ONE LP MATHEMATICAL MODEL OF UNKNOW MIXTURES WITH LIMITED PROPERTIES AND FIXED MIXTURES AT PRODUCTS MAKING WITH AVAILABLE RESOURCES (THE EXAMPLE OF HEAT-RESISTING MATERIALS)*)

Nikolić I. , Božilović S.[1,2], Koprivica S.[1], Todorović I.[2], Giorgio A.[3]
[1]Faculty for Construction Management Studies, Cara Dušana 62, Belgrade, Serbia
[2]Faculty of Enterprenuership, Cara Dušana 62, Belgrade, Serbia
[3]Studio di Ingeneria Informatica, CTU Tribunale di Milano, Via Roma 89, Milano, Italia

Abstract: *This study presents mathematical problem modeling of mixing available resources and making proper products having all required demands and characteristics in accordance with the limitations regarding raw materials, technical capacities and market of product sales. Optimization of the carried out total profit is performed and two types of products have been considered and analyze: (a) with unknown mixtures of raw materials and imposed limits for needed product characterstics and (b) imposed mixtures of raw materials defining needed product characteristics. Making selected types of heat-resisting concrete meant for panelling in thermal plants is presented.*
Keywords: *optimization, mixture problem, combined mixture-production problem, heat-resisting materials, shuttering, thermal plants*

Intruduction

In the literature dealing with operational research, or in other words – quantitive methods, there is well-known problem of the mixture relating to the problems of linear programming. There is a request saying that combination of certain materials defines a product having required characteristics representing the quality of the product itself, which is significant for the purpose of that product. On of the most common examples is that one showing defining diet nutritition meals, but there is also various examples of industrial products. Each type of raw materials contains the mentioned characteristics in certain amount (some characteristics may not be present) so that the concrete mixture, i.e. the product itself, is provided with required characteristics. It is common that the price of mixture is minimized for a product unit. Characteristics of raw materials are either known or defined by adequate analyses. There are necessary limitations for the realized characteristics within a product: low and upper limits are to be applied to all characteristics or certain limits for some of characteristics (low limits or upper limits only). [1] - [4]

Certainly, more than one mixture can be analyzed at the same time, that is – it is possible to analyze more products and minimize the total price of needed materials. In this case, the problem of more mixtures is expanded by making final products in accordance with the planned period limited factors. Then, there follows the classification of such problems and they are divided into three problems and the third one is presented to be the combination of another two ones. Making heat-resisting concrete for shuttering in thermal plants is illustrated beginning with [4] i [5]. What is presented is described as products ranging in accordance with their profitability. Special attention is paid on the possibility of potential applying of multi-criteria optimization and desired programminge (theoretical assumptions of general models and methods have been presented in [6] - [10]). Selected examples have been shown in [11] - [15]: multi-mixtures problem (the first problem in the following classification),

*) This study is supported by Ministry of Science and Technology Development of the Republic of Serbia. The first research phase on Project TR 36023 “The new trends of design and management on construction industry”, 2011-2014.

multi-criteria selection of heat-resisting materials for special purpose, defining product profitability for the common production issue and applying desired programming to production planning.

Mixture and making products with available resources

The problem of mixture for multi-products can be expanded by making certain products and analyzing existing limitations for raw materials, technical capacities and product position at the market within proper planned period. Also, there may appear three types of problems: (1) the lack of known material mixtures for products and the presence of certain limits for necessary characteristics, (2) mixtures of raw materials are defined for products, and (3) combination of problems (1) and (2). In case of analyzing the mixtures for more products simultaneously in (1) and (3), a minimal total price of the needed materials may not match the sum of minimal raw material prices within partial problems for individual products. The problem 1 is presented in [4] i [5], and the problem (2) in [4].

Mathematical model for problem (3)

Two groups of products are being analyzed by taking their mixtures into consideration. The first group includes the products having unknown mixtures and required limits for certain characteristics that are to be realized by mixing certain raw materials. The second group includes the products having known mixtures for needed raw materials. In order to make products, limited quantities of raw materials and available technical capacities are used. Maximization of profit with product quantities accepted by the market within the planned period is required.

In order to define general mathematical model of the defined problem, it is suitable to use the following parameters: p = product number P_k; $k \in K=\{1,2,...,p\}$; z_k = unknown quantity P_k ($k \in K$); $K^1 \subset K$ = set of production indexes for p_1 where there are no known ways of mixing raw materials; $K^2 \subset K$ = set of indexes for p_2 product having known ways of mixing raw materials ($p_1<p$, $p_2=p-p_1$ i $K^1 \cap K^2=\emptyset$, and also $K^2=K \backslash K^1$); m = number of characteristics A_i for mixtures/products P_k from the first group with $k \in K^1$; $i \in I=\{1,2,...,m\}$; n = number of raw materials S_j; $j \in J=\{1,2,...,n\}$; $J_k \subset J$ = set of indexes for n_k raw material S_j that are used for the product P_k ($k \in K$); a_{ij} = contents of A_i within unit S_j ($i \in I$; $j \in J$); a_{ik}^L, a_{ik}^U = low border (LB) and upper border (UB) for A_i in P_k from the first group ($i \in I$, $k \in K^1$); x_{jk} = unknown quantities of S_j for the quantity z_k of the product P_k from the first group ($j \in J_k$, $k \in K^1$); b_{jk} = proportional participation of S_j in P_k from the second group ($j \in J_k$, $k \in K^2$); c_j = price for the unit S_j ($j \in J$); v = number of r resources G_r that are used in production; $r \in R=\{1,2,...,v\}$; h_{rk} = normatives of consumption G_r for the unit P_k ($r \in R$, $k \in K$); h_r = available capacities G_r in the planned period ($r \in R$); b_j = available quantity S_j in the planned period ($j \in J$); d_k = selling price for the unit P_k; K_1, K_2, K_3, K_4 = subsets of the product indexes P_k having the following conditions for quantities z_k in accordance with sale: e_k^L = low border; e_k^E = fixed quantity ($e_k^L = e_k^U$); e_k^U = upper border. As for the sets of indexes of the product, it is obvious that the following is applied: $K_1 \cup K_2 \cup K_3 \cup K_4 = K$ i $K_s \cap K_\beta=\emptyset$ ($s,\beta=1,2,3,4$; $s \neq \beta$), where $k \in K_1$ includes only low borders, $k \in K_2$ only fixed quantities, $k \in K_3$ only upper borders and $k \in K_4$ both low and upper borders. It implies only the production connected with market or the mentioned borders for production quantities including proper stoks as well. Suitable common mathematical model has the following characteristics:

$$\text{(max)} \quad D_1(z,x) = \sum_{k \in K^1} \left(d_k z_k - \sum_{j \in J_k} c_j x_{jk} \right) + \sum_{k \in K^2} \left(d_k - \sum_{j \in J_k} c_j b_{jk} \right) z_k \qquad (1)$$

subject to:

$$a_{ik}^{L} z_k \le \sum_{j \in J_k} a_{ij} x_{jk} \le a_{ik}^{U} z_k \text{; } i \in I; k \in K^1 \quad (2)$$

$$\sum_{j \in J_k} x_{jk} = z_k \text{; } k \in K^1 \quad (3)$$

$$\sum_{k \in K^1} x_{jk} + \sum_{k \in K^2} b_{jk} z_k \le b_j \text{; } j \in J_k, k \in K \quad (4)$$

$$\sum_{k \in K} h_{rk} z_k \le h_r \text{; } r \in R \quad (5)$$

$$t_{jk}^{1,L} \le x_{jk} \le t_{jk}^{1,U} \text{; } j \in J_k; k \in K^1 \quad (6)$$

$$t_{jk}^{2,L} \le b_{jk} z_k \le t_{jk}^{2,U} \text{; } j \in J_k; k \in K^2 \quad (7)$$

$$z_k \begin{cases} \ge e_k^L; & k \in \{K_1 \cup K_4\} \\ = e_k^E; & k \in K_2 \\ \le e_k^U; & k \in \{K_3 \cup K_4\} \end{cases} \quad (8)$$

$$x_{jk} \ge 0 \text{; } j_k \in J; k \in K^1 \quad (9)$$

$$z_k \ge 0 \text{ and integers; } k \in K \quad (10)$$

Function of criterion $D_1(x,z)$ in (1) presents the difference between realized income and product sale and raw materials costs, which claims maximal value. This value represents Profit 1 (or Benefit 1). In order to define actual profit it is necessary to include all types of expenses. Limitations define the process of carrying out the following demands: (2) characteristics A_i with the given borders within products P_k from the first group; (3) necessary incoming quantities x_{jk} of raw materials S_j for outgoing quantities z_k of product P_k from the first group; (4) total usage of S_j for the products from the first and second group within the borders of available quantities; (5) total involvement of G_r for all the products in accordance with available capacities; (6), (7) participation of S_j in P_k from the first group, that is the second group, depending on the imposed borders; (8) unknown quantities of the product P_k with allowed borders; (9)-(10) natural conditions for unavailable elements including potential requiring that the quantities of P_k are integers.

It is necessary to point out that, in accordance with the analogy of borders analyses for products in (8), borders for the quantities of raw materials S_j for certain products P_k in (6) and (7) can also be analyzed, as well as the borders for A_i characteristics in P_k product from the first group in (2). Furthermore, a total physical volume of production, q = Q(z), is possible to be defined as the sum of quantities z_k.

Illustrative example

A part of the production range of the company ″Rudnici i industrija šamota Aranđelovac″ is being examined by using the example p=8 products of heat-resisting concrete (Z-1 to Z-8); p_1=3 products from the first group with the indexes $K^1=\{1,4,7\}$; p_2=5 products from the second group with $K^2=\{2,3,5,6,8\}$; n=4 type of raw materials (n_1=1 own raw material: S1 – Šamot; and n_2=3 imported raw materials: LC – heat-resisting cement Lafrage, KB – Chinese concrete, S71 – Secar 71 Lafrage); m=1 characteristic for products from the first group (Alumina); v=1 production resource (Mixer) and planned period of 1 month.

One of the special characteristics is that each product is formed by mixing two raw materials: $J_k=\{1,2\}$ for k=1,2,...,6; $J_7=\{2,3\}$ and $J_8=\{3,4\}$. For the products of the first group both types of borders are set (6) for A_1 characteristic and raw materials S_j; $j \in J_k$, $k \in K^1$. (3) leads to tho following: it is enough to limit raw material LC with j=2 that takes part in every P_k with $k \in K^{1,}$ which defines the borders for another raw material in pair (as an addition up to 100% in suitable mixture).

Table 1: Data for raw materials and the first group of production

Raw materials S_j		j=1, S1	j=2, LC	j=3, KB	j=4, S71
Alumina (A_1), a_{1j}		35.00%	40.00%	85.00%	70.00%
unknowns, [t] borders for S_j in P_k, %	Z-1, z_1, x_{j1} [$t^{1,d}$; $t^{1,g}$]	x_{11} –	x_{21} [34.90%; 35.10%]	–	–
	Z-4, z_4, x_{j4} [$t^{1,d}$; $t^{1,g}$]	x_{14}	x_{24} [29.90%; 31.10%]	–	–
	Z-7, z_7, x_{j7} [$t^{1,d}$; $t^{1,g}$]	–	x_{27} [69.90%; 70.10%]	x_{37} –	–

Table 2: Data for the first group of products

Products, P_k		Z-1	Z-4	Z-7
Alumina (A_1)	LB, t_{1j}^L UB, t_{1j}^U	36.50% 37.50%	36.50% 37.50%	75.00% 76.00%
Mixer (G_1), h_{1j} [hour/t]		0.50	0.50	0.50
Selling price, d_k [monetary units/t]		585.00	488.00	691.00
Unknowns, z_k [t]		z_1	z_4	z_7
Sale borders		e_k^L=50.00[t] and e_k^U=100.00[t], $k \in K$		

Table 3. Data for the second group of products

Products, P_k		Z-2	Z-3	Z-5	Z-6	Z-8
Raw materials (S_j), b_{jk} [%/t]	j=1, S1	65%	70%	75%	80%	0
	j=2, LC	35%	30%	25%	20%	0
	j=3, KB	0	0	0	0	80%
	j=4, S71	0	0	0	0	20%
Mixer (G_1), h_{1j} [hour/t]		0,50	0,50	0,50	0,50	0,50
Selling price, d_k [m.u./t]		515.00	492.00	491.00	499.00	978.00
Unknowns, z_k [t]		z_2	z_3	z_5	z_6	z_8
Sale borders		e_k^L=50.00[t] and e_k^U=100.00[t], $k \in K$				

Table 4. Additions for resources

Resources		Prices, c_j [m.u./t]	Available, b_j, h_1
Raw materials, S_j	j=1, S1	500.00	800.00 [t]
	j=2, LC	300.00	150.00 [t]
	j=3, KB	250.00	200.00 [t]
	j=4, S71	1000.00	150.00 [t]
Mixer, G_1			352.00 [hour]

Model solving and solutions analyses

Maximal Profit 1 is D_1*=135,363.00[m.u.]. Optimal values for the raw material quantities that are used for the first group of products are: x_{11}*=65.95[t], x_{21}*=35.05[t], x_{12}*=34.95[t], x_{22}*=15.05[t], x_{27}*=80.00[t] i x_{37}*=20.00[t]. Optimal quantities for five products are suitable for the upper borders of sale z_k*=100[t] za k=1,2,6,7,8. In case of other products, low borders of sale have been realized z_k*=50[t] for k=3,4,5.

Taking into consideration the consumption of raw materials b_{jk} and optimal quantities z_k for the second group of products, needed quantities of suitable raw materials can be defined

$b_{jk} \cdot z_k$* (Table 5): 0.65·100 = 65.00[t] of raw materials S1 and 0.35·100 = 35.00[t] of raw materials LC for z_1=100[t] product Z-2 etc.

Defining product profitability

The above presented problem solving type does not provide the answer to the main issues dealing with business efficiency. (a) Are there the products that are supposed to be produced in larger quantities in comparison to the requested upper borders, since they do possess more profitability and can lead to the growth of total profit $D_1(x,z)$? (b) Are there the products that are supposed to be produced in smaller amounts than the requested low borders, since they do possess less profitability and they lead to total profit decrease $D_1(x,z)$?

During the production optimization it is necessary to define certain products profitability in the process of gradual implementing needed borders for adequate products. [4] [14]

Step1) At the beginning one should define a solution 1) without analyzing borders for selling products. It has been confirmed that z_1^{1*}=429[t] and z_8^{1*}=250[t] realize maximal profit D_1^{1*}=210,965.00[m.u.], and other products are not available. (Table 6). Products Z-1 and Z-8 have higher profitability. It is said that such products possess Rang 1 in accordance with the analyzed criterion $D_1(x,z)$, which can be described as r_1=1 and r_8=1.

Step 2) Setting the upper borders $z_1 \leq 100$[t] i $z_8 \leq 100$[t] defines the solution 2) with $z_1^{2*}=z_8^{2*}$=100[t], z_2^{2*}=243[t], z_7^{2*}=150[t] and D_1^{2*}=158,595.00[m.u.]. The products Z-1 and Z-8 with $r_1=r_8=1$ from the solution 1) possess the values regarding the implemented upper borders, and new products in this solution Z-2 i Z-7 possess Rang 2 ($r_2=r_7=2$). There is a significant total profit decrease and what has been achieved is D_1^{2*}=75,176%D_1^{1*}.

Steps 3)–5) Further borders setting for products with the quantities that are larger than the mentioned borders, that is – setting the borders for new products in the last solution (step) with positive quantities and keeping the borders for the previously analyzed products (in all previous steps), defines their profitability rank order in accordance with adequate steps taken from solution process. Solution 3) points out r_6=3 for Z-6, and solution 4) concludes r_3=4 for Z-4. Since the solution 5) realizes z_5^{5*}=40.00[t] and z_4^{5*}=0[t], it leads to the conclusion that Z-5 has r_5=5 and Z-4 has r_4=6.

It turns out that the considered n=8 products within the exposed problem can be divided into 6 groups of profitability with the following lexicography order: (Z-1, Z8) > (Z-2, Z7) > Z-3 > Z-6 > Z-5 > Z-4. By analyzing the values for new positive unknowns in certain solutions, a more detailed ranging of certain products can be carried out. In the solution 1) z_1^{1*}=429.00[t] > z_8^{1*}=250.00[t]. That is why Z-1 possesses higher profitability than Z-8 and so r_1=1 for Z-1 and r_2=2 for Z-2. The solution 2) with z_2^{2*}=243.00[t] > z_7^{2*}=150.00[t] leads to r_2=3 and r_7=4. As a result: r_3=5, r_6=6, r_5=7 and r_4=8, that is – total rang list of the products: Z-1 > Z8 > Z-2 > Z7 > Z-3 > Z-6 > Z-5 > Z-4.

Steps 6)–5) Solution 5) has z_4^{5*}=0 and z_5^{5*}=40[t] with smaller values than the low borders 50[t]. It is necessary to set law borders for these products. Having low border for z_4 in the solution 6), the quantity of product Z-5 is z_5^{6*}=0, Z-4 has the requested quantity z_5^{6*}=50[t] and Z-3 is decreased from z_3^{5*}=100[t] to the acceptable value z_3^{6*}=83[t] that is higher than low border. In case there are two simultaneously set low borders for z_4 and z_5, there comes the solution 7) having further quantity decreasing of Z-3 to unacceptable value z_3^{7*}=42[t] below low border. Also, the quantity of Z-7 is slightly decreased as well, from its upper border to the acceptable value z_7^{7*}=99[t] above the upper one. Finally, when the low border has been set for z_3 as well, it leads to the solution 8) with the quantities for four products on the upper borders (k=1,2,7,8), for three products on low borders (k=3,4,5) and for one product within the given borders (k=6). Such solution has been defined at the beginning when the needed borders were set simultaneously for all the products.

Certainly, any other more specific limitation is going to make the previously realized value for optimized criterion worse, D_1^1* from the solution 1) is realized in the following solutions with the percentages: 75.176%, 64.750%, 64.341%, 64.295%, 64.246%, 64.188%, 64.164%. The presented example shows that additionally calculated value for the total product quantity is also decreased, Q(z), which does not happen in a general case.

Table 5: Solutions 1) – 4) with the conditions for the unknowns z_j (j=1,8,2,7,6)

	Solutions	1)	2)	3)	4)
Solution elements		$z_k \geq 0, k \in K$	$z_1, z_8 \leq 100$	$z_2, z_7 \leq 100$	$z_6 \leq 100$
S1, x_{11}	[t]	279.00	65.05	65.00	64.90
LC, x_{21}	[t]	150.00	34.95	35.00	35.10
S1, x_{14}	[t]	0	0	0	0
LC, x_{24}	[t]	0	0	0	0
LC, x_{27}	[t]	0	30.00	20.00	20.00
KB, x_{37}	[t]	0	120.00	80.00	80.00
z_1	[t]	429.00	100.00	100.00	100.00
z_2	[t]	0	243.00	100.00	100.00
z_3	[t]	0	0	0	133.00
z_4	[t]	0	0	0	0
z_5	[t]	0	0	0	0
z_6	[t]	0	0	300.00	100.00
z_7	[t]	0	150.00	100.00	100.00
z_8	[t]	250.00	100.00	100	100.00
$D_1(x,z)$	[m.u.]	210,965.00	158,595.00	136,600.00	135,736.00
Q(z)	[t]	679	593	700	633
S1, b_1=800.00	[t]	279.000	223.00	370.00	303.00
LC, b_2=150.00	[t]	<u>150.000</u>	<u>150.00</u>	<u>150.00</u>	<u>150.00</u>
KB, b_3=200.00	[t]	<u>200.000</u>	<u>200.00</u>	160.00	160.00
S71, b_4=150.00	[t]	50.000	20.00	20.00	20,00
Mixer, h_1=352.00	[hour]	339.900	296.50	350.00	316.50

The solution 8) after restrictons in next order: $z_3 \leq 100$, $z_4 \geq 50$, $z_4 \geq 50$ and $z_3 \geq 50$

$x_{11} = 64.95$, $x_{21} = 30.05$, $x_{14} = 34.95$, $x_{24} = 15.05$, $x_{27} = 20.00$, $x_{37} = 80.00$
$z_1 = z_2 = 100$, $z_3 = z_4 = z_5 = 50$, $z_6 = 87$, $z_7 = z_8 = 100$
$D_1(x,z) = 135{,}363.00$; $Q(z) = 637$
$b_1^* = 307.00$, $b_2^* =$ <u>150.00</u>, $b_3^* = 160.00$, $b_4^* = 20.00$, $h_1^* = 318.50$

Using resources capacities and production bottleneck

Let's mark raw material usage as b_j^* (j=1,2,3,4) and drying –room involvement as h_1^* in the solution for the basic problem with given borders for product quantities. It is obvious that only available quantity of raw material is completely used (100%), since $b_2^*=b_2=150$[t]. Using raw materials is $b_1^*/b_1 = 307.00/800.00 = 38.375\%$ for S1, $b_2^*/b_2=150/150=100\%$ for LC, $s_3^*/b_3 = 160.00/200.00 = 80{,}000\%$ for KB $s_4^*/b_4 = 20.00/150.00 = 13{,}333\%$ for S71. Resources engagement G_1 (Mixer) is $h_1^*/h_1 = 318.50/352.00 = 90{,}483\%$.

As for profitability, it is neccesary to point out that the ranked products, which have been defined in the solutions 1) to 9) for the initial raw material quantities and resource G_1, do not have to be the same if the solution 1) begins with b_2^9 from the solution 9), that is with b_2^{10} and h_1^{10} from the solution 10). Product ranging is carried out by using constant starting capacities. The solution 9) shows that Z-6 has the lowest rang r_6=8, but in case of b_2^8 the rang is r_6=6.

Raw material mixtures for the first group of products

Defined model of the analyzed problem confirms that the first group of P_k products ($k \in K^1$), for which there are no specified raw material mixtures S_j ($j \in J_k$), do not have constant mixtures in all presented solutions. Allowed borders for A_1 characteristic within those products define necessary raw materials in accordance with the specified criterion – total profit is to be maximized and certain borders for selling products are to be defined. Namely, optimal value for the criterion $D_1(x,z)$ defines adequate optimal quantities of x_{jk} raw materials S_j within the first group of P_j products in certain solutions. For example, Z-1 with z^1_1*=429.00[t] in the solution 1) uses x^1_{11}*/z^1_1* = 279.00/429.00 = 65.035% of raw material S1 and x^1_{21}*/z^1_1* = 150.00/429.00 = 34.965% of raw material LC. Other solutions possess constant value z_1*=100[t], but S1 is used from 64.95% to 65.50% and LC is used from 34.50% to 35.10%, or in other words – in the needed amount up to 100%.

Conclusion

This study deals with the issues of available material mixtures needed for the process of forming more products. The mentioned problem has been broadened by making products for the plan period and analyzing the borders that are to be applied to available resources (raw materials, technical capacities) as well as selling products limitations. Classification of the mentioned problems have been presented (unknown mixtures, known mixtures, combinations of both) as well as mathematical modelling of the combined problem of mixing raw materials needed for making two groups of products in order to perform total profit optimization. Raw materials possess adequate characteristics and the mixture of such materials provides necessary characteristics for the products, in terms of quality and purpose. As for the first group of products, a model is used in order to form mixtures on the basis of raw material characteristics and borders for the needed characteristics of these products. In case of the second group of products, raw material mixtures are specified.

The mentioned defined model has been tested in the situation of making selected types of heat-resisting concrete belonging to one national Company having its own raw materials as well as foreign materials. It turned out that the same product from the first group does not necessarily possess the constant mixture of raw materials in some solutions. Also, there is a process of products ranging on the basis of product profitability from [4] and [14].

Applying the models of multi-criteria optimization and goal programming is very significant (for theoretical assumptions and methods see [6] – [10]). For example, [15] presents the models of linear goal programming to accomplish required profit with the following tasks: (a) needed product quantities with available resource capacities and (b) needed product quantities and needed resource capacities.

References

[1] L. Lapin, *Quantitative Methods for Business Decision with Cases*, Harcourt Brace Jovanovich - Publishers, Academic Press, San Diego, 1975. (Fourth Edition 1998.)

[2] B. Render, and R. M. Stair Jr., *Quantitative Analysis for Management*, Allyn and Bacon, Inc., Boston, 1982. (Second Editinon, 1985.)

[3] S. Krčevinac and all., *Operation Research (Ореарциона истраживања),* Belgrade State University, Faculty of Organization Sciences, Belgrade, 2004. (in Serbian)
[4] I. Nikolić, and S. Božilović, *Quantitatve Metods and Models for Management – Selected problems in Construction Company – Application of WinQSB and Expert Choice Software (Kvantitativne metode i modeli u menadžmentu – Odabrani problemi u graditeljstvu – Primena softvera WinQSB i Expert Choice),* University "Union", Faculty for Construction Management Studies, Belgrade, 2009. (in Serbian)
[5] A. Terzić, and Lj. Pavlović, *Construction Materials for Heat-Resisting usage (Konstrukcioni materijali za visokotemperaturnu primenu),* ITNMS, Belgrade, 2009. (in Serbian)
[6] Hwang C. L. And A. S. Masud, *Multiple Objective Decision Making – Methods and Applications,* Lecture Notes in Economics and Mathematical Systems, Springer-Verlag, Berlin, Heidelberg, New York, 1979.
[7] I. Nikolić, and S. Borović S., *Multicriteria optimization – Methods, Logistics Applications and Software (Višekriterijumska optimizacija* - *metode, primena u logistici, softver)*; Center of Military Schools, Belgrade, 1996. (in Serbian)
[8] Ehrgott Matthias, *Multicriteria optimization,* Springer-Verlag Berlin Heidelberg, 2000.
[9] Lee S. M., *Goal Programming for Decision Analysis*, Auerbach Publischers Inc., Philadelphia, 1972.
[10] Ignazio J. P., *Goal Programming and Extensions*, Lexington Books, Massachusetts, 1976.
[11] I. Nikolić, S. Koprivica, and Z. Cekić, "The optimization of mixture problem on the example of heat-resisting materials", *Communications in Dependability and Quality Management – An International Journal*", Volume 12, Number 3, September, 2009, p. 4-12
[12] I. Nikolić, and Z. Cekić, "Multicriteria Analysis of The Heat-Resisting Materials Selection on Specific Project" (Višekriterijumska analiza za izbor vatrostalnog materijala na specifičnom projektu), *Proceding, YUPMA 2009., XIII Internacional symposium for Project Management,* Zlatibor, Serbia, 6-8 june 2009., p. 369-373 (in Serbian)
[13] Z. Cekić, and I. Nikolić, "Knowledge Base of heat-resisting materials", *Proceding, ICDQM 2009, 12th International Conference Depedability and Quality Management,* Belgrade, Serbia, 25-26 June 2009, p. 665-673
[14] M. Draškić, and I. Nikolić, "Ranking of products on assortment optimization by LP application", "*Copper" - Magazine published by the business association of organizations of associated labor involved in copper and copper-basec products production, fabrication and marketing "Jugobakar",* No. 40, Yugoslavia, 1985., p. 30-33
[15] I. Nikolić, "Extensive Applications of Linear Goal Programming for Optimization of Production Assortment" (Široke mogućnosti primene ciljnog linearnog programiranja u izboru asortimana proizvodnje), *Proceding, Simposium ETAI '85,* Ohrid, Macedonia, 1985., p. 576-583 (in Serbian)

Mathematical model

Firstly, one should calculate the costs of raw materials $\Sigma c_j b_{jk}$ for the product unit P_k from the second group ($k \in K^2$) in order to define the values $d_{1,k} = d_k - \Sigma c_j b_{jk}$ on the basis of the selling prices $d_{k,}$ for Profit 1 in the second added of the criterion function (1).

$$\begin{aligned}(\max)\ D_1(x,z) = \ & -500x_{11} \quad -300x_{21} \quad +585z_1 \quad +85z_2 \quad +52z_3 \\ & -500x_{14} \quad -300x_{24} \quad +488z_4 \quad +41z_5 \quad +39z_6 \\ & -300x_{27} \quad -250x_{37} \quad +691z_7 \quad +578z_8\end{aligned}$$

Limitations for the first group of products

		S1	LC	Z-1	S1	LC	Z-4		
Z-1	A_1, LB ...	$0.35x_{11}$	$+ 0,40x_{21}$	$- 0.365z_1$				$\geq$	0
	A_1, UB ...	$0.35x_{11}$	$+ 0.40x_{21}$	$- 0.375z_1$				$\leq$	0
	Raw materials...	x_{11}	$+ x_{21}$	$- z_1$				=	0
	LC, LB ...	x_{11}		$- 0.349z_1$				$\geq$	0
	LC, UB ...	x_{11}		$- 0.351z_1$				$\leq$	0
Z-4	A_1, LB ...				$0.35x_{14}$	$+ 0.40x_{24}$	$- 0.365z_4$	$\geq$	0
	A_1, UB ...				$0.35x_{14}$	$+ 0.40x_{24}$	$- 0.375z_4$	$\leq$	0
	Raw materials...				x_{14}	$+ x_{25}$	$- z_4$	=	0
	LC, LB ...				x_{14}		$- 0.299z_4$	$\geq$	0
	LC, UB ...				x_{14}		$- 0.231z_4$	$\leq$	0

		LC	KB	Z-7		
Z-7	A_1, LB ...	$0.85x_{27}$	$+ 0.70x_{37}$	$- 0.81z_7$	$\geq$	0
	A_1, UB ...	$0.55x_{27}$	$+ 0.70x_{37}$	$- 0.82z_7$	$\leq$	0
	Raw materials...	x_{27}	$+ x_{37}$	$- z_7$	=	0
	LC, LB ...	x_{14}		$- 0.299z_4$	$\geq$	0
	LC, UB ...	x_{14}		$- 0.231z_4$	$\leq$	0

Limitations for resources

	Z-1	Z-2	Z-3	Z-4	Z-5	Z-6	Z-7	Z-8		
S1 ...	x_{11}	$+ 0.65z_2$	$+ 0.70z_3$	$+ x_{14}$	$+ 0.75z_5$	$+ 0.80z_6$			$\leq$	800.00
LC ...	x_{21}	$+ 0.35z_2$	$+ 0.30z_3$	$+ x_{24}$	$+ 0.25z_5$	$+ 0.20z_6$	$+ x_{27}$		$\leq$	150.00
KB ...							$+ x_{37}$	$+ 0.80z_8$	$\leq$	200.00
S71 ...								$+ 0.20z_8$	$\leq$	150.00
G_1 ...	$0.5z_1$	$+ 0.5z_2$	$+ 0.5z_3$	$+ 0.5z_4$	$+ 0.5z_5$	$+ 0.5z_6$	$+ 0.5z_7$	$+ 0.5z_8$	$\leq$	352.00

Sale limitations

$$50 \leq Z_k \leq 100;\ k=1,2,...,8$$

natural limitations for unknowns

$$x_{jk} \geq 0\ (j \in J_k,\ k=1,4,7),\ z_k \geq 0 \text{ and integers } (k=1,2,...,8)$$